題號： 301
科目：工程數學(I)
節次： 6

國立臺灣大學 107 學年度碩士班招生考試試題

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1. (a) **(15%)** Find a real general solution to the following homogeneous system of DEs:

$$\begin{bmatrix} y'_1 \\ y'_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

(b) **(10%)** Determine the type and stability of the critical point. Then sketch or graph three of the trajectories in the phase plane. Show the details of your work.

(c) **(5%)** Find the particular solution by the following initial condition.

$$\begin{bmatrix} y_1 (0) \\ y_2 (0) \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

2. **(10%)** Using the method of undetermined coefficients, find a particular solution to the following

$$y'' + 9y = \sin \left(3x + \pi/4 \right)$$

3. **(10%)** Solve the initial value problems by Laplace transform.

$$y'' + y = f(t), \qquad y(t = 0) = 0, \qquad y'(t = 0) = 0$$

where $f(t) = 0$ if $t < 2$ and $f(t) = t$ if $t \geq 2$

4. (a) **(5%)** Calculate eigenvalues and eigenvectors of matrix A.

$$A = \begin{bmatrix} 2.8 & 1.2 \\ 1.2 & 2.8 \end{bmatrix}$$

(b) **(10%)** Calculate $A^{10}y1$ and $A^{10}y2$.

$$y1 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \ y2 = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{bmatrix}$$

(c) **(10%)** An elastic membrane in the (x_1, x_2) plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched by A^2: $y = A^2 x$. Calculate and plot the original boundary circle and new boundary under deformation.

5. **(25%)** Solve the heat equation in an infinite bar. Show every step of your calculation. Plot $u(x, t)$ for $t = 0, 1, 2, 5$ sec.

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$u(x, 0) = \begin{cases} 50°C, & \text{if } |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$c^2 = 1 \left(\frac{cm^2}{sec} \right)$$

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