1. Let $X_1, X_2, \ldots, X_n$ be i.i.d. Poisson distribution with mean λ.

   (a) (5 points) Find the moment-generating function of X_1.

   (b) (10 points) Find the uniformly minimum variance unbiased estimator (UMVUE) of $(\lambda - 1)(\lambda - 2)$.

   (c) (10 points) Let X be the Poisson distribution with mean λ and the conditional distribution of Y given $X = x$ be the binomial distribution with size x and probability p. Show that the marginal distribution of Y is the Poisson distribution with mean $p\lambda$.

2. (10 points) Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from the uniform distribution on the interval $(0, \theta)$ with an unknown parameter $\theta \in (1, \infty)$. Suppose that we only observe

$$X_i = \begin{cases} Y_i & \text{if} \quad Y_i \geq 1 \\ 1 & \text{if} \quad Y_i < 1 \end{cases}$$

for $i = 1, \ldots, n$. Find a moment estimator of θ.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution on the interval $(\theta, \theta + |\theta|)$. Find the maximum likelihood estimator (MLE) of θ when

   (a) (5 points) $\theta \in (0, \infty)$.

   (b) (5 points) $\theta \in (-\infty, 0)$

   (c) (5 points) $\theta \in \mathbb{R}, \ \ \theta \neq 0$.

4. Let $X_1, X_2, \ldots, X_n$ denote a random sample from $N(\theta, 36)$.

   (a) (5 points) Obtain the likelihood ratio test for testing $H_0 : \theta = \theta'$ versus $H_1 : \theta \neq \theta'$, where θ' is a specified constant.

   (b) (5 points) Is the test derived in (1a) a uniformly most powerful test?

5. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ denote the order statistics of a random sample $X_1, X_2, \ldots, X_n$ from the uniform distribution $U(\theta, \theta + 1)$, where θ is a real number.

   (a) (10 points) Does the family of all possible joint distributions of $(X_{(1)}, X_{(n)})$ have monotone likelihood ratio?

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(b) (10 points) When the nominal level is set at α, find a uniformly most powerful test for testing $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$.

6. Let $X_1, X_2, \ldots, X_{n_1}$ and $Y_1, Y_2, \ldots, Y_{n_2}$ denote independent random samples from the normal distributions $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, respectively. The sample means and sample variances are denoted by $\bar{X}$, $\bar{Y}$, S_1^2 and S_2^2, respectively.

   (a) (15 points) Show that the distribution of random quantity

$$Q = \frac{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

   can be approximated by a chi-squared distribution, and obtain its corresponding effective degrees of freedom δ.

   (b) (5 points) Assume that $\sigma_1^2 \neq \sigma_2^2$, the following test statistic

$$t = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

   is frequently used for assessing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$. Show that the distribution of t can be approximated by a t-distribution with δ degrees of freedom.

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