1. (25%) Solve the initial value problem by Ordinary Differential Equations (ODEs).

$$y'' + y' - 2y = 0, \quad y(0) = 5, \quad y'(0) = -4.$$

2. (20%) Determine the Laplace transform of the function $f(t)$ that is periodic and defined on one period as follow.

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ t-2, & 1 \le t < 2 \end{cases}$$

3. (30%) If there are some positive integers p making $\mathbf{A}^p = \mathbf{0}$, then $\mathbf{A}$ is said as the nilpotent matrix.

(a) Show that a nilpotent matrix is necessarily singular,
(b) If $\mathbf{A}$ is nilpotent, with $\mathbf{A}^p = \mathbf{0}$, show that

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots + \mathbf{A}^{p-1}$$

(c) Find the inverse of the given matrix.

$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & 7 & 1 \end{bmatrix}$$

4. (25%) Let $x(t)$ be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of the following signal in terms of a_k.

$$x(t - t_0) + x(t + t_0)$$

試題隨卷繳回