1. (30%) Referring to Figure 1, a platform moves in the horizontal direction on the ground surface. A motor Q is placed on the platform, driving a rod with a length of L and a negligible mass. A ball of mass m is attached to the end of the rod. Suppose the system is initially at rest, then the motor drives the rod with a constant angular acceleration of α when the platform begins to move with a constant velocity of V_o.

(1) (5%) Please derive the velocity of the ball when $\theta = \pi/2$.

(2) (5%) Please calculate the maximum achievable height of the ball if the rod is broken when $\theta = \pi/2$.

(3) (5%) If the rod is broken when $\theta = \pi$, find the distance of the ball from the origin O when it hits the ground.

(4) (5%) Continued from (3), please compute the maximum rebounded height if the coefficient of restitution between the ball and the ground is *e*.

(5) (10%) Continued from (4), please estimate the distance of the ball from the origin O when it hits the ground the n-th time (suppose the coefficient of restitution between the ball and the ground is *e*). What is its theoretical maximum travel distance?

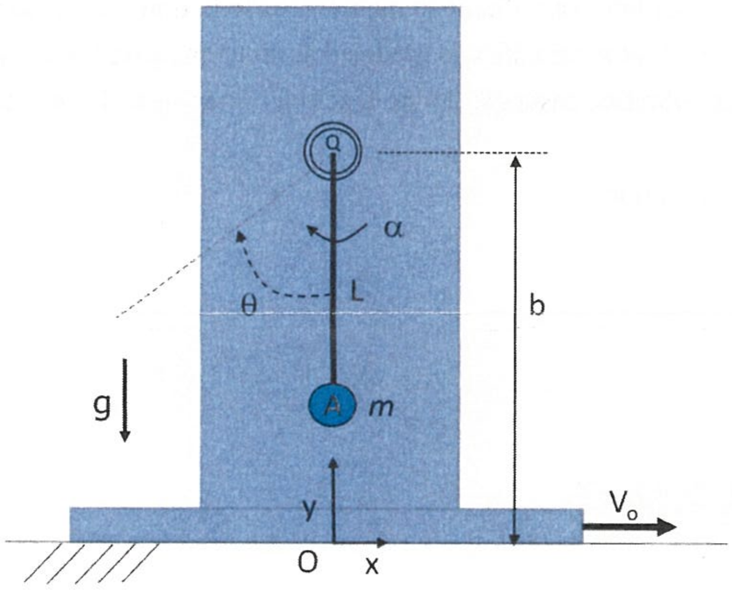

Fig. 1

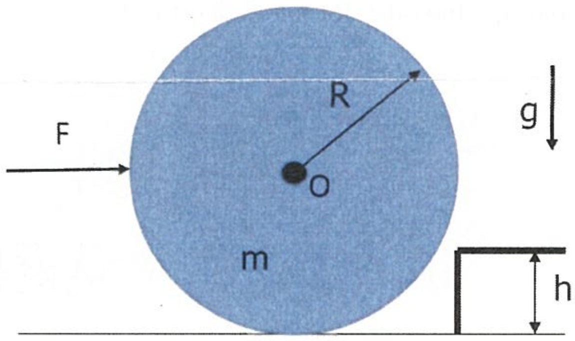

Fig. 2

2. (20%) Referring to Figure 2, a uniform solid disk with a mass of m and a radius of R is placed on a surface. The coefficient of friction between the disk and the surface is μ. The disk is initially at rest before a constant horizontal force F is applied and pointed to the center of the disk.

(1) (5%) Please estimate the maximum allowable force F such that the disk rotates without slipping.

(2) (5%) Please compute the corresponding acceleration of the disk given the critical force obtained in (1).

(3) (10%) The disk travels a distance of L before hitting a corner with a height of h=R/2. Please calculate the minimum distance of L such that the disk can roll over the corner.

3. (30%) A uniform, rigid rod is initially positioned on a frictionless surface as shown in Fig. 3, and then it is released from that position at t = 0. The length of the rod is **L**, and the mass of the rod is **m**. The angle between the rod and the horizontal plane is $\boldsymbol{\theta}$. Also, the moment of inertial about the mass center G of the rod is $\mathbf{I_G}$.

(1) (5%) Draw the freed-body diagram and define the coordinate systems.

(2) (8%) Express the equation of motion for the falling rod in terms of **L**, **m**, $\boldsymbol{\theta}$, and $\mathbf{I_G}$.

(3) (10%) Assume that L = 2 m and m = 10 kg. Find the angular velocity $\dot{\boldsymbol{\theta}}$ associated with $\boldsymbol{\theta} = 30°$ when the rod is released from rest at $\boldsymbol{\theta} = 60°$.

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(4) (7%) In question (3), you have determined the angular velocity $\dot{\theta}$ when $\theta = 30°$. How do you determine when the rod reaches $\theta = 30°$?

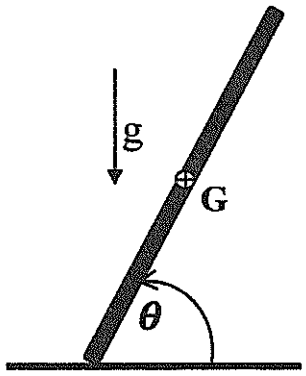

Fig. 3

4. (20%) A machine's support springs will compress 0.6 cm under the weight of the machine when it is put on the floor. Based on this information, we want to approximate the ratio of the machine's oscillation amplitude to the floor's oscillation amplitude if the floor is vertically vibrating at a frequency ω (rad/s). Suppose that the vibratory machine is modeled as an undamped, linear system with a sinusoidal input, and we are most concerned with the forced vibration response of the machine. The mass of the machine is **m** and the lumped spring stiffness is **k**.

(1) (8%) Model the vibratory system and derive the equation of motion.

(2) (12%) Determine the ratio for $\omega = 80\pi$ (rad/s).

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