1. (20%) Consider a stationary inertial frames of reference O and a non-inertial frame of reference O' moving with a time-varying velocity $V(t)$. The coordinate transformation between two frames of reference is

$$x' = x - \int_0^t V(\tau)\, d\tau \ ,$$

$$t' = t \ ,$$

where x and x' are position vectors in frames O and O', respectively.

a) (15%) Use the chain rule to show that the material derivative is Galilean invariant, i.e.,

$$\frac{D}{Dt'} = \frac{\partial}{\partial t'} + \boldsymbol{u}' \cdot \nabla' = \frac{\partial}{\partial t} + \boldsymbol{u} \cdot \nabla = \frac{D}{Dt} \ ,$$

where $\boldsymbol{u}$ and $\boldsymbol{u}'$ are velocity vectors in frames O and O', respectively

b) (5%) Using the result from part (a), verify that

$$\boldsymbol{a}' = \frac{D\boldsymbol{u}'}{Dt'} = \frac{D\boldsymbol{u}}{Dt} - \frac{d\boldsymbol{V}}{dt}(t) = \boldsymbol{a} - \frac{d\boldsymbol{V}}{dt}(t) \ ,$$

where $\boldsymbol{a}$ and $\boldsymbol{a}'$ are acceleration vectors in the frames O and O', respectively.

2. (30%) Consider a lifting flat plate at a non-zero angle of attack in an incompressible, inviscid flow as shown in figure 1.

a) (4%) Briefly state the d'Alembert's paradox associated with incompressible, inviscid flow.

b) (4%) Draw a replication of figure 1 on your answer sheet. Based on the statement of part (a), sketch the direction of the resultant force on the replicated figure.

c) (4%) We can decompose the resultant force into a component normal to the flat plate and a component tangent to the flat plate. Explain the source of the normal component of the resultant force.

d) (9%) The tangential component of the resultant force is due to a phenomenon called ``leading edge suction." The leading edge suction is due to the low pressure field generated by the accelerating flow past around the leading edge, which can be modeled by a potential flow past around a sharp edge as shown in figure 2. The corresponding velocity potential ϕ of this potential flow in the polar coordinate (r, θ) is

$$\phi = C\ r^{1/2} \cos\left(\frac{\theta}{2}\right) \ ,$$

where C is a constant. If the pressure at infinity is P_∞ and the density of the fluid is ρ, determine the pressure field generated by this velocity potential as a function of the distance to the leading edge, r.

e) (9%) By integrating the gauge pressure from part (d) around the sharp edge, show that although the pressure is infinite as $r \to 0$, the corresponding force is tangent to the flat plate and is finite.

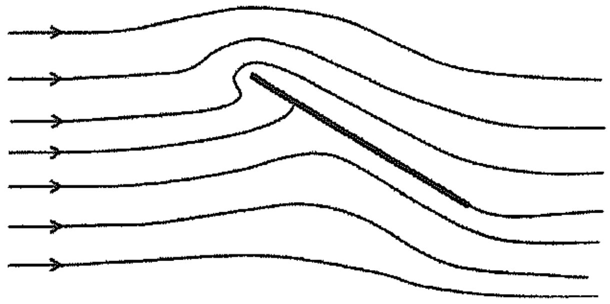

figure 1 for problem 2.

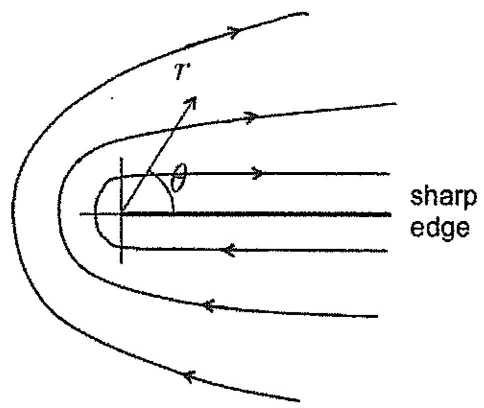

figure 2 for problem 2.

3. (40%) A long belt is pulled vertically out of a reservoir at constant velocity U so that a thin liquid layer of thickness h is drawn away with it on both sides of the belt as sketched in figure 3. Assume the reservoir is very large that the pulling does not disturb the stationary liquid sufficiently far away. Liquid density and viscosity are ρ and μ, respectively, and gravitational acceleration is g.
   a) (30%) Solve for the stress and viscous heating at segment A and C.
   b) (10%) Discuss how you may solve the stress at segment B and compare its magnitude to those at segment A and C with reasons (for example, stress at B is greater than stress at A because .....).

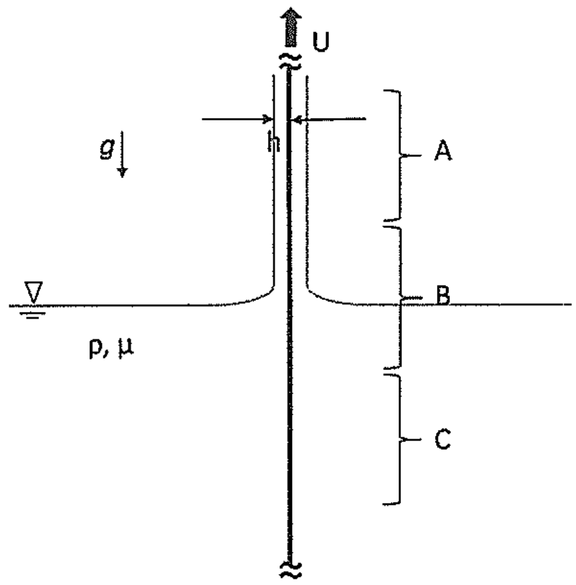

Figure 3 for problem 3.

4. (10%) Wind blows over a shallow stationary liquid layer of thickness h to cause a small disturbance to its height into h+Δh and this small displaced liquid propagates at velocity *a*, as shown in figure 4. Liquid density and viscosity are ρ and μ, respectively, and gravitational acceleration is g. Use dimension analysis to find a scaling relation for *a*.

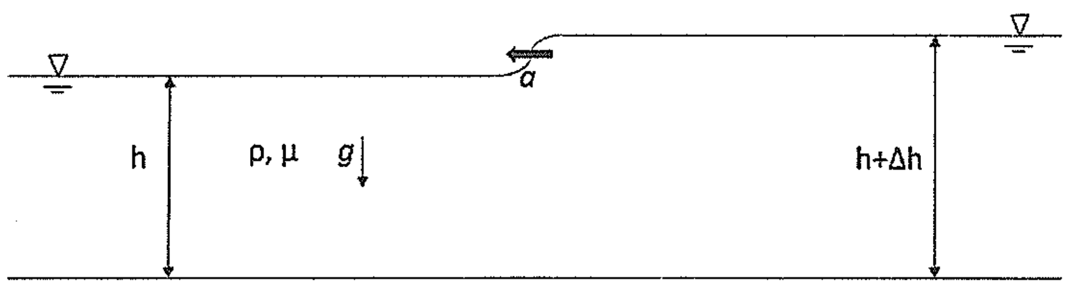

Figure 4 for problem 4.

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