題號： 104
科目：微積分(D)
節次： 1

國立臺灣大學 107 學年度碩士班招生考試試題

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※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (20%) Let $f(x) = x + x^3 + x^4, x \geq 0$ and let $g(x)$ be the inverse function of $f(x)$.
   (1) Find $g'(3)$ and $g''(3)$.
   (2) Evaluate
   $$\int_0^3 g(x)\, dx.$$
   (3) Evaluate
   $$\int_0^9 g(\sqrt{x})\, dx.$$

2. (20%)
   (1) Let $f(x) = xe^x$. Find $\lim_{h \to 0} \dfrac{f(x+4h) + f(x) - 2f(x + 2\ln(1+h))}{h^2}$.
   (2) Let g be twice differentiable. Show that there exists $z \in [x-1, x+1]$ such that $g(x+1) - 2g(x) + g(x-1) = g''(z)$.

3. (20%)
   (1) Show that $\lim_{N \to \infty} \dfrac{1 + 1/2 + 1/3 + \cdots + 1/N}{\ln N} = 1$.
   (2) Let $1 \leq a_k \leq 2$ and $S_k = \sum_{i=1}^{k} a_i$. Show that $\lim_{N \to \infty} \dfrac{\sum_{k=1}^{N} \dfrac{a_k}{S_k}}{\ln S_N} = 1$.

4. (20%) Find the point(s) on the ellipsoid $x^2 + 4y^2 + z^2 = 14$ where $x - 4y + 3z$ attais its maximum value.

5. (20%) Find the surface area of the sphere $x^2 + y^2 + z^2 = 9$ inside the cylinder $x^2 + y^2 = 3x$. (Hint: use polar coordinates)

試題隨卷繳回