

1. A uniform plane wave traveling in vacuum has only y and z components of electric field strength of the

$$\text{form } \begin{cases} E_y = 5 \sin \left[ \omega \left( t - \frac{x}{c} \right) + \frac{\pi}{6} \right] \text{ (V/m)} \\ E_z = 5 \sin \left[ \omega \left( t - \frac{x}{c} \right) - \frac{\pi}{3} \right] \text{ (V/m)} \end{cases}, \text{ where } \omega \text{ and } c \text{ are the angular frequency of the wave and}$$

the speed of light in vacuum, respectively.

- (1) What is the polarization state of this uniform plane wave? (4 points)

[The polarization state must be one of the following: linear, right-handed elliptical, left-handed elliptical, right-handed circular, or left-handed circular polarization.]

- (2) Find the time-average Poynting vector of the wave in  $\text{W/m}^2$ . (6 points)

2. Assume that a non-magnetic medium has  $\epsilon = 10\epsilon_0$  and  $\sigma = 0.1 \text{ mho/m}$ , where  $\epsilon_0 = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}$ . A

uniform plane wave of 100 MHz propagates in this medium in the x direction. Find the ratio of the electric field at  $x = 0$  to that at  $x = 0.1 \text{ m}$  and the phase difference (in degrees) of the electric field between the two points. (5 points each) Also, find the ratio and phase difference (in degrees) of the MAGNETIC field between the two points. (4 points)

3. The static potential distribution  $V$ , referenced at  $r_b$ , about an infinite length cylinder along the z axis and

of radius  $r_0$  is written as  $V(r) = \frac{10^{-2}}{2\pi\epsilon} \ln\left(\frac{r_b}{r}\right)$  (V) for  $r_0 \leq r \leq r_b$ . Find the volume charge density in the specified region. (6 points)

$$[\text{In cylindrical coordinates, } \nabla A = \hat{\rho} \frac{\partial A}{\partial \rho} + \hat{\phi} \frac{1}{\rho} \frac{\partial A}{\partial \phi} + \hat{z} \frac{\partial A}{\partial z} \text{ and } \nabla^2 A = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial A}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 A}{\partial \phi^2} + \frac{\partial^2 A}{\partial z^2}]$$

4. Given the electric current distribution  $\vec{J}$  in vacuum ( $\epsilon_0, \mu_0$ ), please write down the associate time-dependent Maxwell's equations in differential form in terms of the electric field intensity and magnetic flux density. (8 points)

5. (1) A charged particle of mass  $m$  and charge  $q$  is initially moving with a velocity  $v$  in a uniform magnetic field  $B_0$  that is pointing into the page as shown in Fig. 1(a). It is known that the particle's trajectory is circular. Find the radius of the circle  $R$  and the associated angular frequency  $\omega_c$ . (3 points each)

(2) Assume that an electron ( $m = 9.1 \times 10^{-31} \text{ kg}$  and  $q = -1.6 \times 10^{-19} \text{ C}$ ) with a velocity of  $2 \times 10^7 \text{ m/s}$  enters a magnetic field of  $\vec{B}_0 = \hat{z}(-5 \times 10^{-4}) \text{ webers/m}^2$  as shown in Fig. 1(b). The magnetic field is assumed to be restricted to a 4-cm long region. Find the point of exit ( $x_1, y_1$ ) from the magnetic field region. The electron will follow a straight line after the exit. (6 points)

6. Fig. 2(a) shows the schematic of a transmission line terminated by an unknown load ( $Z_L$ ), and the voltage standing wave pattern of Fig. 2(a) is plotted as Fig. 2(b).

(a) What is the distance between successive voltage minima in Fig. 2(b)? (express in term of the guided wavelength of the transmission line,  $\lambda_g$ ) (5 points)

(b) The maximum and minimum voltages in Fig. 2(b) are 0.3 and 0.1 V, respectively. The distance of the first voltage minimum from the load is  $\lambda_g/8$ . Derive the reflection coefficient ( $\Gamma$ ) at  $d = 0$ . (7 points)

(c) Calculate the value of  $Z_L$ , where  $Z_0$  is  $50 \Omega$ . (3 points)

7. Fig. 3 shows the complex reflection coefficient ( $\Gamma$ ) plane. Derive the corresponding normalized impedance of the thick line shown in Fig. 3. (8 points)
8. In the system shown in Fig. 4, find the characteristic impedance ( $Z_1$ ) of Line 2 and Line 3 for which there is no reflection at the input ( $\Gamma = 0$ ), where  $\lambda_g$  is the guided wavelength of the transmission line. (7 points)
9. For an air-filled rectangular waveguide with dimension of  $a = 5$  cm,  $b = 2.5$  cm. Find the frequency range which only allows single mode propagation in this waveguide. (8 points)
10. TE modes are excited in an air dielectric parallel-plate waveguide of dimension  $a = 5$  cm by setting up at its mouth a field distribution have

$$E = 10(\sin 20\pi x + 0.5 \sin 60\pi x) \sin 10^{10} \pi t \mathbf{a}_y.$$

Determine the propagation mode(s) and obtain the expression for the electric field of the propagating wave. (12 points)

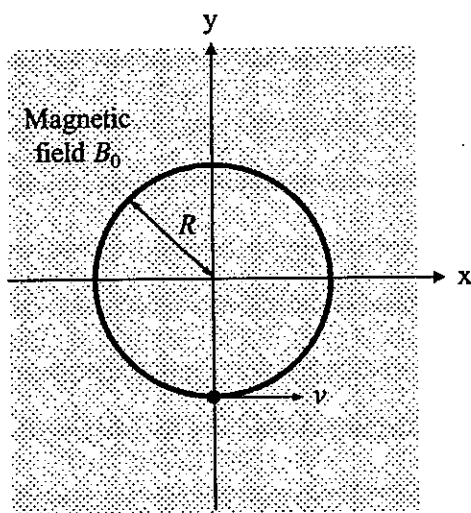


Fig. 1(a)

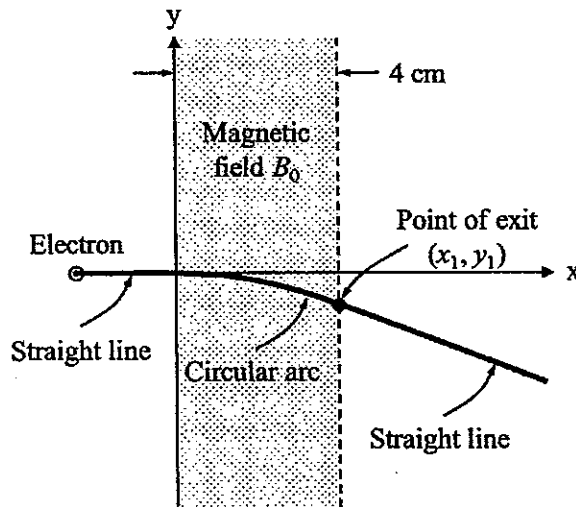
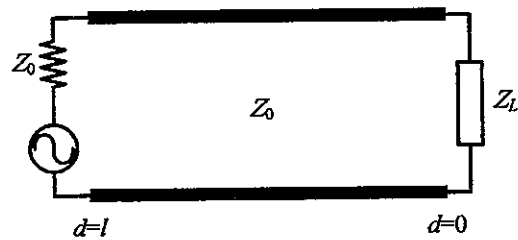
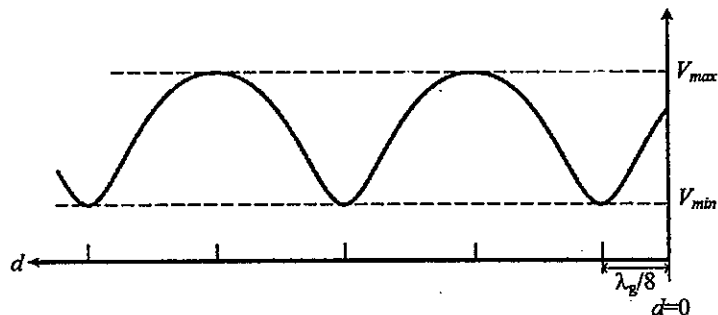


Fig. 1(b)



(a)



(b)

Fig. 2

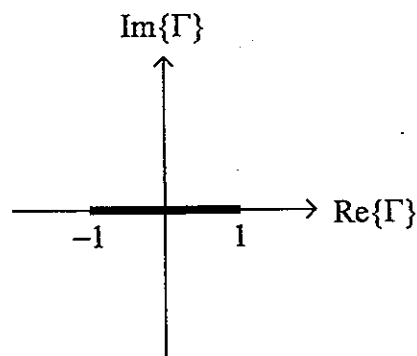


Fig. 3

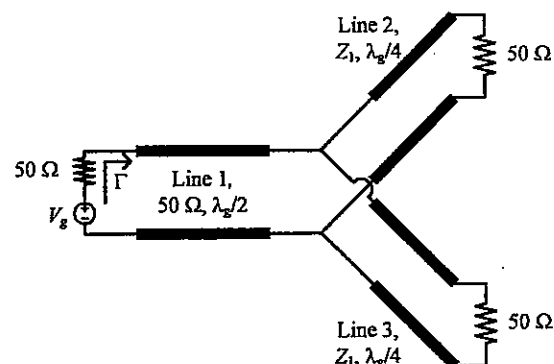


Fig. 4