1. (20 %)

Consider the following system of ODEs:

$$\begin{bmatrix} y_1' \\ y_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos(2x) \\ 0 \end{bmatrix}$$

Derive a particular solution by using the method of variation of parameters. The final solution should be real.

2. (10 %)

Find the general solution

$$-e^x \cos(y) - 4x + [e^x \sin(y) + 3]y' = 0$$

3. (20 %)

Solve the initial value problems by Laplace transform

(a) $y'' + y = 2 + e^t + \sin(2t),\ y(t=0) = 1, y'(t=0) = 0$

(b) $y'' - 5y' + 4y = \delta(t-2),\ y(t=0) = 0, y'(t=0) = 1$

4. (15 %)

The $x_1 - x_2$ plane is stretched by factor 9 and 5 in the principal directions of $[\frac{\sqrt{3}}{2}, \frac{1}{2}]$ and $[-\frac{1}{2}, \frac{\sqrt{3}}{2}]$, respectively. The point (y_1, y_2) of the new coordinate system is stretched from the point (x_1, x_2) of the old coordinate system, given by $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Please calculate $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$.

5. (15 %)

In a Cartesian coordinate $x - y - z$, assume the vector field is

$$[v_1, v_2, v_3] = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}]$$

where f is a scalar function. Is this vector field a rotational or irrotational field? Show your proof.

6. (20 %)

Solve the one-dimensional heat equation $\frac{\partial u}{\partial t} = (\frac{\partial^2 u}{\partial x^2})$ in a bar of length $L = 10$. The initial temperature is $f(x) = x$ if $0 < x < \frac{L}{2}$ and $f(x) = (L - x)$ if $\frac{L}{2} < x < L$. The ends of the bar are kept at temperature 0. Find the solution of the temperature. Please show every step of your work.

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