題號： 255
科目：線性代數(B)
節次： 8

國立臺灣大學 106 學年度碩士班招生考試試題

題號： 255
共 1 頁之第 1 頁

1. True or false, with reason if true and counterexample if false: (15%)

    (a) If $L_1U_1 = L_2U_2$ (upper triangular U's with nonzero diagonal, lower triangular L's with unit diagonal), then $L_1 = L_2$ and $U_1 = U_2$. The LU factorization is unique.

    (b) If $A^2 + A = I$ then $A^{-1} = A + I$.

    (c) If all diagonal entries of A are zero, then A is singular.

2. Find the symmetric factorization $A = LDL^T$ of (15%)

$$A = \begin{bmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix}$$

3. Suppose the matrices in $PA = LU$ are (15%)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -3 & 2 \\ 2 & -1 & 4 & 2 & 1 \\ 4 & -2 & 9 & 1 & 4 \\ 2 & -1 & 5 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 4 & 2 & 1 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

    (a) What is the rank of A?

    (b) What is a basis for the row space of A? (choose vectors in A)

    (c) What is a basis for the column space of A? (choose vectors in A)

    (d) What is the dimension of the left nullspace of A?

    (e) What is the general solution to $Ax = 0$?

4. Use Gram-Schmidt to construct an orthonormal pair q_1, q_2 from $a_1 = (4, 5, 2, 2)$ and $a_2 = (1, 2, 0, 0)$. Express a_1 and a_2 as combination of q_1, and q_2, and find the triangular R in $A = QR$. (15%)

5. Find a straight line which gives the best fit to the curve $y = x^3$ between $x = 0$ and $x = 1$ by using orthogonality. (20%)

6. Solve the second-order equation $\dfrac{d^2\vec{u}}{dt^2} = \begin{bmatrix} -3 & -1 \\ -1 & -3 \end{bmatrix} \vec{u}$ $\vec{u}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and $\vec{u}'(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. (20%)

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