

※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. (10%) Two thin-walled pressure vessels, one spherical and the other cylindrical, with equal inner radius  $r = 1$  m are both subjected to internal pressure  $p = 10$  MPa. The vessels are made of the same material with Young's modulus  $E = 250$  GPa, the shear modulus  $G = 100$  GPa, and the allowable shear stress  $\tau_{allow} = 100$  MPa. Determine (and compare) the thicknesses required for the two vessels.
2. (15%) A cantilever bar (beam/shaft) with channel section of equal flanges is subjected to transverse loads. The equal-flange channel member is known to have one plane of symmetry.
  - (a) (6%) Plot diagrams showing the distributions (magnitudes and directions) of the normal and shear stresses when the channel is so placed that its plane of symmetry coincides with the loading plane of the transverse loads. Are there shear stresses due to torsion?
  - (b) (9%) Plot diagrams showing the distributions (magnitudes and directions) of the normal and shear stresses when the channel is so placed that its plane of symmetry is perpendicular to the loading plane of the transverse loads. Are there shear stresses due to torsion?
3. (20%) A horizontal bar (rod/beam)  $\{ (x, y, z) \mid 0 \leq x \leq 10 \text{ m}, -0.2 \leq y \leq 0.2 \text{ m}, -0.4 \leq z \leq 0.4 \text{ m} \}$  with length  $L = 10$  m and cross-sectional area  $0.4 \times 0.8 \text{ m}^2$  is made of a material with Young's modulus  $E = 100$  GPa and the coefficient of thermal expansion  $\alpha = 10 \times 10^{-6}/^\circ\text{C}$  in the upper half portion  $z \leq 0$  and of another material with Young's modulus  $2E$  and the coefficient of thermal expansion  $2\alpha$  in the lower half portion  $z \geq 0$ . The bar is fixed at the left end  $x = 0$  and is free at the right end  $x = 10$  m.
  - (a) (10%) Suppose the bar is subjected to a concentrated right-directed axial load  $P = 10$  kN applied at the point  $(x, y, z) = (10, 0, 0)$  m. Find the  $z$ -distributions of the normal strains  $\epsilon(x = 5, y = 0, z)$  m and the normal stresses  $\sigma(x = 5, y = 0, z)$  on the vertical cross-section plane  $x = 5$  m and also the  $x$ -distributions of the shear stresses  $\tau(x, y = 0, z = 0)$  on the horizontal plane  $z = 0$  of the interface between the upper and lower portions. Find the longitudinal (horizontal) displacement  $u(10)$  and the transverse (vertical) deflection  $w(10)$  at the right end  $x = 10$  m.
  - (b) (10%) Suppose the entire bar is subjected to a uniform temperature rise  $\Delta T = 50^\circ\text{C}$ . Find the  $z$ -distributions of the normal strains  $\epsilon(x = 5, y = 0, z)$  m and the normal stresses  $\sigma(x = 5, y = 0, z)$  on the vertical cross-section plane  $x = 5$  m and also the  $x$ -distributions of the shear stresses  $\tau(x, y = 0, z = 0)$  on the horizontal plane  $z = 0$  of the interface between the upper and lower portions. Find the longitudinal (horizontal) displacement  $u(10)$  and the transverse (vertical) deflection  $w(10)$  at the right end  $x = 10$  m.
4. (25%) A horizontal cantilever beam has length  $L = 10$  m ( $0 \leq x \leq 10$ ) and bending rigidity  $EI = 6$  GN-m<sup>2</sup> and is subjected to a uniformly distributed load  $q = 3$  kN/m acting vertically downward over the left half span  $0 \leq x \leq 5$  m. The beam is free at the left end  $x = 0$  and fixed at the right end  $x = 10$  m.
  - (a) (10%) Find the reactions and plot the shear and moment diagrams. Find the deflection  $w(0)$  at  $x = 0$ .
  - (b) (15%) Suppose the beam has an additional pin support at  $x = 5$ . Find the reactions and plot the shear and moment diagrams. Find the deflection  $w(0)$  at  $x = 0$ .
5. (30%) An angle section of unequal legs has the following geometric properties:  $\int_A dA = 14.00 \text{ cm}^2$ ,  $\int_A Y dA = 22.00 \text{ cm}^3$ ,  $\int_A Z dA = 43.00 \text{ cm}^3$ ,  $\int_A Y^2 dA = 74.61 \text{ cm}^4$ ,  $\int_A Z^2 dA = 244.5 \text{ cm}^4$ ,  $\int_A YZ dA = 28.91 \text{ cm}^4$  when referred to a rectangular Cartesian coordinate system OYZ.
  - (a) (10%) Locate the centroid  $o$  of the cross section (and translate the OYZ system to a parallel  $oyz$  system). Orientate the principal axes through the centroid and determine the principal moments of inertia of the cross section.
  - (b) (20%) A vertical column, prismatic with the above unequal-leg angle section, is fixed at the lower end  $x = 0$  and otherwise nowhere supported. The column has Young's modulus  $E = 200$  GPa and is subjected to a compressive load  $P$  at the upper end  $x = 60$  cm acting through the centroid. Formulate the relation between the bending moments, shear forces, and  $P$ . Find the buckling load  $P_{cr}$ , the buckled direction, and the buckled shape.