國立臺灣大學 105 學年度碩士班招生考試試題

1. (20 pts) Find the general solution $(x(t), y(t))$ of the system

$$\begin{cases} y'(t) + x(t) = 0, \\ x''(t) - y(t) = 0. \end{cases}$$

2. (20 pts) Let $x(t)$ is a C^1 function on $\mathbb{R}$ with $x(0) = 0$.
(a) Prove that $x(t) = 0$ for all t if $0 \le x'(t) \le 2x(t)$.
(b) Prove that the same conclusion $x(t) = 0$ for all t holds under the weaker assumption $-2|x(t)| \le x'(t) \le 2|x(t)|$.

3. (20 pts)
(a) Suppose $x' - x = h(t)$, $x(0) = 0$. Find a function $A(t)$ such that

$$x(t) = \int_0^t A(t-s)h(s)\,ds.$$

(b) Suppose $y'' + 2y = h(t)$, $y(0) = 0, y'(0) = 0$. Find a function $B(t)$ such that

$$y(t) = \int_0^t B(t-s)h(s)\,ds.$$

4. (20 pts)
(a) Show that $\tanh'(t) = 1 - \tanh^2(t)$.
(b) Let $x(t) = p + q\tanh(\frac{t}{2})$. Find $p, q, c \in \mathbb{R}$ such that $x'' + cx' + 2x(x - \frac{3}{4})(1 - x) = 0$.

5. (20 pts) Let $x(t)$ satisfy $x'' - x^3 + x = 0$.
(a) Show that $\frac{1}{2}[x'(t)]^2 - \frac{1}{4}x^4(t) + \frac{1}{2}x^2(t) =$ constant.
(b) Assume $x(0) = 1, x'(0) = 0$. Find $x(t)$.

試題隨卷繳回