

(答案請寫於答案卷上)
需列計算過程,否則不予計分

填充計算題(總計10題,每題10分)

- Two sequences of positive integers $\{a_n\}$ and $\{b_n\}$ are defined recursively by taking $a_1 = b_1 = 1$ and equating rational and irrational parts in the equation $a_n + b_n\sqrt{2} = (a_{n-1} + b_{n-1}\sqrt{2})^2$ for $n \geq 2$.
Find the value of $a_n^2 - 2b_n^2 =$ _____ (10 points)
- Let $a_n = n^n/n!$. Find the value of $\lim_{n \rightarrow \infty} \frac{n}{(n!)^{1/n}} =$ _____ (10 points)
- Find the value of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} =$ _____ (10 points)
- Is the product of $\sum_{n=0}^{\infty} (-1)^{n+1}/\sqrt{n+1}$ with itself a divergent series? Show your work, otherwise, no credit is given. (10 points)
- Evaluate the value of $\int_1^{\infty} \frac{\log x}{x^{n+1}} dx =$ _____ (10 points)
- Let $f(x)$ be a differentiable function for $x \geq 0$ and $f(0) = 0$. If $\int_0^x \sqrt{1 + f'(t)^2} dt = (1 + 2x)^{3/2}$. Then $f(x) =$ _____ (10 points)
- The volume generated by revolving the curve $y = \cos x, x \in [0, \frac{\pi}{2}]$ around x-axis is _____ (10 points)
- Determine the interval of t such that $\int_1^{\infty} \frac{\ln x}{x^t} dx$ is convergent. Also, the associated value of $\int_1^{\infty} \frac{\ln x}{x^t} dx$ is _____ (10 points)
- Evaluate the value of $\int_1^{\frac{\pi}{2}} (\sin x)^3 (\cos x)^{28} dx =$ _____ (10 points)
- The value of $\int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$ where $x^{2/3} + y^{2/3} = 1$ is _____ (10 points)

試題隨卷繳回