Show all the details of your work.

1. (10%)

Find the general solution.

$y' + ysinx = e^{cosx}$

2. (15%)

Find the particular solution.

$x^2y''-4xy'+6y=0, \quad y(1)=0.4, y'(1)=0$

3. (10%)

Find the Laplace Transform. Show the details.

$(a) \quad (a-bt)^2$

$(b) \quad e^{3t}sinht$

4. (15%)

Given an ODE $y''+3y'+2y=10\sin t$. Find the general solution by Laplace transform.

5. (25%)

Solve the heat equation by Fourier series. Assume there is a laterally insulated bar with temperature 0 at both end; the initial temperature of the bar is

f(x)=x if 0<x<5

f(x)=(10-x) if 5<x<10

Derive two ordinary differential equations (ODEs) from the heat equation. Then solve ODEs with boundary and initial conditions. Show the final solution of the problem.

6. (25%)

Determine eigenvalues and eigenvectors of the following matrix.

$$A = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix}$$

An elastic membrane with boundary $x_1^2 + x_2^2 = 1$ is stretched by the following equation such that (x_1, x_2) goes to (y_1, y_2). Plot the original boundary, new boundary, and principal stretch directions.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 5/2 & 1/2 \\ 1/2 & 5/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

試題隨卷繳回