1. Find the general solutions of the differential equations:

   (1) $x^2y'' - 2xy' + 2y = 0$ (10%)

   (2) $y'' - 4y' + 5y = e^{2x}\csc x$ (10%)

2. Find the solution of $x'' + 2x' + 2x = g(t)$, $x(0) = 0$, $x'(0) = 0$
   Where, g(t) = 1 for $0 \le t \le 1$, g(t) = 0 for t > 1, by Laplace transform. (15%)

3. Write down the "Divergence theorem" and "Stokes's theorem" in vector analysis, and with adequate explanation (better with the aid of figures). (15%)

4. Consider a 1.0×10^7 m^3 lake fed by a polluted stream having a flow rate of 5.0 m^3/s and pollution concentration equal to 10.0 mg/L. There is also a sewage outfall that discharges 0.5 m^3/s of wastewater having a pollutant concentration of 100 mg/L. The stream and sewage wastes have a reaction rate coefficient of 0.20/day. Assuming the pollution is completely mixed in the lake, and assuming no evaporation or other water losses or gains, (a) find the steady-state pollutant concentration. (10%)

   If we divert the sewage outfall around the lake, eliminating it as a source of pollution, assuming complete-mix conditions, (b) find the concentration of pollution in the lake one week after the diversion. (15%)

5. One way to disaggregate environmental impacts of human activity is with the following product:

   Impacts = (Population) x (Affluence) x (Technology)

   Where, affluence is indicated by per capita energy demand, while technology is represented by the carbon emission per unit of energy. Each factor is itself growing exponentially; that is, the rate of change of the quantity N is proportional to N ($dN/dt = rN$). The proportionality constant r is called the rate of growth and has unit of ($time^{-1}$). If per capita energy demand increases at 1.5% per year, fossil fuel emissions of carbon per unit of energy decreases at 0.5% per year, and population grows at 1.0% per year, (a) how long would it take before we are emitting carbon at twice the current rate? (15%)

   (b) At that point, by what fraction would per capita energy demand have increased? (10%)

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