1.　Factor A into LU, and find the solution for: (20%)

$$A\vec{x}=\begin{bmatrix} 2 & -1 & & \\ -1 & 2 & -1 & \\ & -1 & 2 & -1 \\ & & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 3 \\ -1 \end{bmatrix}$$

2.　Which of the following are subspaces of R^{∞} with reason? (20%)

(a) All sequences like $(1, 0, 1, 0,\ldots)$ that include infinitely many zeros.

(b) All sequences $(x_1, x_2,\ldots)$ with $x_j = 0$ from some point onward.

(c) All decreasing sequences: $x_{j+1} \leqq x_j$ for each j.

(d) All convergent sequences: the x_j have a limit as $j \rightarrow \infty$.

(e) All geometric progressions $(x_1, kx_1, k^2x_1,\ldots)$ allowing all k and x_1.

3.　Find a best approximation to $y = x^5$ by a straight line between x = -1 and x = 1. (20%)

4.　Find orthogonal vectors A, B, C by Gram-Schmidt process from $a = (1, -1, 0, 0)$, $b = (0, 1, -1, 0)$, $c = (0, 0, 1, -1)$. (10%)

5.　Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 0 & -i & 0 \\ i & 1 & i \\ 0 & -i & 0 \end{bmatrix}. \text{ (10%)}$$

6.　$A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ and show A is positive definite. If $\vec{x}^T A \vec{x} = 1$, draw the tilted ellipse

and find the half-lengths of its axes. (20%)

試題隨卷繳回