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國立臺灣大學105學年度碩士班招生考試試題

科目：微積分(B)

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1 Evaluate  $dy/dx$  and  $d^2y/dx^2$  at the point  $(2, -1)$  of the curve  $x^2 + 4xy + y^3 + 5 = 0$ .

(10%)

2 Find the critical numbers, points of inflection, and vertical and horizontal asymptotes of

$f(x) = x^2(x-7)^{1/3}$  (if any), describe the concavity of  $f$ , and sketch the graph of  $f$ . (20%)

3 Calculate (a)  $\frac{d}{dx}[x^{(2^x)}]$  (b)  $\frac{d}{dx}(\sinh^{-1} x)$  (10%)

4 Evaluate the given integral. (10%)

(a)  $\int_0^{\pi/4} \left[ \frac{1 + \sin x}{\cos^2 x} \right] dx$  (b)  $\int_1^{e^2} x \ln \sqrt{x} dx$

5 State whether the sequence converges and, if it does, find the limit. (10%)

(a)  $a_n = 2 \ln 3n - \ln(n^2 + 1)$  (b)  $a_n = \frac{2^n}{n!}$

6 Determine whether the series converges or diverges.  $\sum_{k=1}^{\infty} \frac{1}{k \ln(k+1)}$  (10%)

7 A solid  $T$  is bounded above by the plane  $z = y$  and below by the paraboloid  $z = x^2 + y^2$ .

Find the volume of  $T$ . (10%)

8 Integrate  $\mathbf{h}(x,y,z) = yz\mathbf{i} + x^2\mathbf{j} + xz\mathbf{k}$  over the line segment from  $(0, 0, 0)$  to  $(1,1,1)$ . (10%)

9 Calculate the total flux of  $\vec{v} = 2x\vec{i} + xz\vec{j} + z^2\vec{k}$  out of the solid bounded by the paraboloid

$z = 9 - x^2 - y^2$  and the  $xy$ -plane. (10%)

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