※ 注意：全部題目均請作答於試卷內之「非選擇題作答區」，請標明題號依序作答。

1. (25%) Consider a homogeneous, isotropic, linearly elastic material.
   (a) (6%) When a material is said to be isotropic, what does it mean? When it is said to be homogeneous, what does it mean? When it is said to be linearly elastic, what does it mean?
   (b) (7%) What is the relation between E, ν, and G? Prove it.
   (c) (6%) Write down the formulae (Hooke's law) expressing the strain components ϵ_{ij} $(i,j=1,2,3)$ in terms of the stress components σ_{ij} $(i,j=1,2,3)$ and Young's modulus E and Poisson's ratio ν.
   (d) (6%) Point out *which relations and which quantities* in the above formulae (Hooke's law) show that they are *linear* and *isotropic*.

2. (20%) A *cylindrical* thin-walled pressure vessel with inner radius $r = 5$ m is subjected to internal pressure $p = 50$ MPa. The vessel is made of a material with Young's modulus $E = 250$ GPa, the shear modulus $G = 100$ GPa, and the allowable shear stress $\tau_{allow} = 100$ MPa.
   (a) (9%) Design (determine) the thickness required. In determining the thickness, do we need to use Young's modulus $E = 250$ GPa and shear modulus $G = 100$ GPa? Why?
   (b) (6%) *Derive* the formulae for computing the three principal stresses. Compute them.
   (c) (5%) Determine the principal strains. Do the principal axes of stresses and strains coincide? Why?

3. (30%) A propped cantilever beam with overhang has length $L = 6$ m $(0 \le x \le 6)$ and bending rigidity $EI = 2$ GN-m^2 and is subjected to a uniformly distributed load $q = 3$ kN/m over $0 \le x \le 2$ m. The beam is free at the left end $x = 0$ and fixed at the right end $x = 6$ m, and has an interior roller support at $x = 2$ m and an interior pin connection at $x = 4$ m.
   (a) (7%) Find the reactions and plot the shear and moment diagrams.
   (b) (4%) Find the slopes $w'(4^-), w'(4^+)$ and the deflection $w(4)$ at the pin connection $x = 4$ m.
   (c) (4%) Find the slope $w'(0)$ and deflection $w(0)$ at the tip of the overhang $x = 0$ m.
   (d) (5%) What is the elastic energy U stored in the beam? What is the work W done by the load $q(x)$? Is $U = W$?
   (e) (2%) Why do we not use the shear modulus G?
   (f) (8%) If there is no interior pin connection at $x = 4$ m, find the reactions and plot the shear and moment diagrams.

4. (25%) A column of length $L = 10$ m $(0 \le x \le 10)$, width $b = 40$ cm $(-20 \le y \le 20)$, and depth $h = 20$ cm $(-10 \le z \le 10)$ is made of a material with Young's modulus $E = 250$ GPa and thermal expansion coefficient $\alpha = 10 \times 10^{-6}/°\text{C}$. Denote the displacements (deflections) in the x, y, z-directions by $u(x), v(x), w(x)$, respectively. Both ends of the column is pin-supported and prevented from moving in the x-direction, $u(0) = u(10) = 0$. The column is heated up uniformly by ΔT in the unit of °C.
   (a) (5%) Determine the axial load P induced in the column when $\Delta T = 30°\text{C}$. Is it tensile or compressive?
   (b) (6%) Plot a free body diagram and formulate the equilibrium equations of *an infinitesimal column element* dx in a deflected-deformed configuration.
   (c) (6%) Derive the governing equation(s) for the deflection(s) of the column and prescribe the boundary conditions and solve for the deflection(s).
   (d) (8%) Determine the buckling load P_{cr}, the critical temperature rise ΔT_{cr} which causes the column to buckle, and the buckled shape $(v(x), w(x))$.

試題隨卷繳回