

1. Find the 3 by 3 matrix A and its pivots, rank, eigenvalues, and determinant:

$$\begin{bmatrix} x & y & z \\ A & x & y & z \end{bmatrix}^T = 4(x - y + 2z)^2. \quad (20\%)$$

2. True or false, with reason if true and counterexample if false: (20%)
- (a) Subspace V orthogonal to W and W orthogonal to Z makes V orthogonal to Z .
- (b) Every nonsingular matrix can be factored into the product $A = LU$ of a lower triangular L and an upper triangular U .
- (c) If A and B are symmetric then AB is symmetric.
- (d) If A and B are invertible then AB is invertible

3. Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 12 \\ 6 \\ 18 \end{bmatrix}$$

- (a) Use the Gram-Schmidt process to find an orthogonal basis for the column space of A .
- (b) Factor A into a product QR , where Q has an orthogonal set of column vectors and R is upper triangular.
- (c) Solve the least squares solution $Ax = b$. (20%)
4. Solve the second-order equation (20%) $\frac{d^2 \bar{u}}{dt^2} = \begin{bmatrix} -5 & -1 \\ -1 & -5 \end{bmatrix} \bar{u}$ $\bar{u}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\bar{u}'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. (20%)
5. Draw the tilted ellipse $x^2 + xy + y^2 = 1$ and find the half-length of its axes. (20%)