1.(20%) For the following matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}, \quad (1)$$

(a)(5%) Compute the determinant det(**A**)=?,
(b)(5%) rank(**A**)=?,
(c)(10%) Find the inverse $\mathbf{A}^{-1}$. Please write the process in detail.

2.(25%) (a) Let $F(s) = \mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}dt$ be the Laplace transform of $f(t)$.
(a1)(3%) $\mathcal{L}[tf(t)]$ =? (In terms of $F(s)$)
(a2)(3%) $\mathcal{L}[\dot{y}(t)]$ =? (In terms of $Y(s)$ and $y(0)$)
(a3)(4%) $\mathcal{L}[\ddot{y}(t)]$ =? (In terms of $Y(s)$, $y(0)$ and $\dot{y}(0)$)
(a4)(5%) Let us consider

$$t\ddot{y}(t) + (1-t)\dot{y} + ny = 0, \quad (2)$$

where $n = 0, 1, 2, \ldots$. Write the solution of $Y(s) = \mathcal{L}[y(t)]$ =? when you apply the Laplace transform to the above equation.
b.(10%) Let us consider

$$y'(x) = \frac{2y - x - 3}{2x - 4y + 5}. \quad (3)$$

Write the general solution.

3.(15%) Let us consider a harmonic oscillator with a stiffness k^2, and under an impulse P at a time $t = 1$:

$$\frac{d^2w(t)}{dt^2} + k^2w(t) = P\delta(t-1), \quad t > 0, \quad k > 0, \quad (4)$$

where δ has the property $\int_0^t f(\xi)\delta(\xi - 1)d\xi = f(1)$ for all $t > 1$. Write the general solution of $w(t)$.

4.(10%) Evaluate $\oint_\Gamma (e^{-x}\cos y dx + e^{-x}\sin y dy)$ by the Green Theorem, where Γ is the rectangle with vertices at $(0,0)$, $(\pi, 0)$, $(\pi, \pi/2)$, and $(0, \pi/2)$.

5.(10%) Find the Fourier series expansion for the following function:

$$f(x) = x^2 - x, \quad -\pi < x < \pi. \quad (5)$$

6.(20%) By using the Fourier series method solve the following partial differential equation:

$$u_t = u_{xx}, \quad 0 < x < 1, \ t > 0, \quad (6)$$
$$u(0,t) = 0, \quad u(1,t) = 2, \quad (7)$$
$$u(x,0) = x^2 + x, \quad 0 \le x \le 1. \quad (8)$$

試題隨卷繳回