1. (16 pts) (a) (6 pts) Describe the graph of the function $f(x) = x^{1/x}$ from $(0, \infty)$. In the answer, you should give where the maximum occurs and increasing and decreasing.
(b) (10 pts) Use (a) to determine the set of pairs of positive real numbers (x, y) with $x^y = y^x$.

2. (15 pts) Define a set of positive real numbers as follows. Let $x_0 > 0$ be any positive number, and let $x_{n+1} = (1 + x_n)^{-1}$ for all $n \geq 0$.

   (a) (5 pts) Prove that $0 < x_n < 1$.

   (b) (10 pts) Determine $\lim_{n\to\infty} x_n$ and justify your answer.

3. (15 pts) Find the surface area of that portion of the surface $z = y^2 + 4x$ which lies above the triangular region R in the xy-plane with vertices at $(0, 0)$, $(0, 2)$ and $(2, 2)$.

4. (10 pts) Evaluate

$$I = \int_0^2 \int_0^{\sqrt{2x-x^2}} \left(k + 3\sqrt{x^2 + y^2}\right) dydx$$

and express your answer in terms of the constant k.

5. (15 pts) Let $\mathbf{F}(x, y, z) = (\exp(x^2) + y)\mathbf{i} + (\sin(y^3) + xz)\mathbf{j} + z^2\mathbf{k}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve $x^2 + y^2 = 10$, $x + y + z = 4$ with positive orientation (i.e., counterclockwise) as viewed from high on the z-axis.

6. (10 pts) Assume that both $f(x)$ and $g(x)$ is differentiable at 2. Evaluate

$$\lim_{x\to 2} \frac{2^n f(x) - x^n f(2)}{2^n g(x) - x^n g(2)} \quad \text{where } n \text{ is a natural number and } \frac{g'(2)}{g(2)} \neq \frac{n}{2}.$$

7. (9 pts) Minimize $(x-2)^2 + 2(y-1)^2$ subject to $x + 4y \leq 3$ and $x \geq y$.

8. (10 pts) Find a real number c so that

$$\left| c - \int_{-1/2}^{1/2} \frac{\exp(x) - 1}{x} dx \right| < 0.01.$$

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