1. The data in the following table represent a random sample generated from the following density

39.1	41.7	45.1	55.5	60.2
53.3	82.9	57.6	41.2	38.5
58.2	50.9	75.2	39.0	36.3
60.1	38.2	45.8	35.9	59.5

$$f_X(x)=\begin{cases}\dfrac{\alpha\omega^{\alpha}}{x^{\alpha+1}}, & x\geq\omega,\quad \alpha>2\\ 0 & x<\omega\end{cases}$$

Find and calculate the method-of-moments estimates of α and ω. **(20%)**

2. A random sample of size 20 $(x_i,\ i=1,\ldots,20)$ from a continuous uniform distribution between 0 and θ (i.e., $U(0,\theta)$) has a maximum value of 28.88 $(\max(x_1,\ldots,x_{20})=28.88)$. Find a 90% confidence interval of θ. **(20%)**

3. Explain the following terms:
   (1) Unbiasd estimator
   (2) Moment generating function
   (3) Poisson distribution
   (4) *p*-value
   (20%)

4. A company is formulating a new product and is interested in foam height (in *mm*). Foam height is approximately normally distributed and has a standard deviation of 20*mm*. The company wishes to test $H_0:\ \mu=180$ *mm* versus $H_1:\ \mu>180$ *mm* using the results of n =16 samples. **(20%)**
   (1) Find the type I error probability α if the critical region is $\bar{x}>190$ *mm*.
   (2) What is the probability of type II error if the true mean foam height is 195 *mm*?
   A few particularly important numbers that are commonly used in significance tests are given in the following table:

p	0.001	0.005	0.010	0.025	0.050	0.100	0.200
Z_p	-3.090	-2.576	-2.326	-1.960	-1.645	-1.282	-0.845

5. Assume two variables (X and Y) are related according to the simple linear regression model, i.e., Y= a + b X. Their quantities are summarized as follows: **(20%)**
   $n=14$, $\Sigma y_i=570$, $\Sigma y_i^2=23{,}500$, $\Sigma x_i=43$, $\Sigma x_i^2=160$, and $\Sigma x_i y_i=1700$.
   (1) Derive the slope (b) and intercept (a) by using the least mean square error method.
   (2) Use the equation of the fitted line to predict *Y* when $x=4.5$.

試題隨卷繳回