國立臺灣大學 102 學年度碩士班招生考試試題

(1) Please solve the following linear ODE (10%)

$x^2y'' + (x^2 + x)y' + (2x - 1)y = 0$

(2) Please solve the following nonhomogeneous ODEs

(a) $y'' - (\frac{1}{x} + 2)y' + (1 + \frac{1}{x})y = (2x + 1 - \frac{1}{x})e^{2x}$ (15%)

(b) $y'' + y = f(t)$, $f(t) = \begin{cases} 1, & t \in (0, \frac{\pi}{2}) \\ \sin t, & t \in (\frac{\pi}{2}, \infty) \end{cases}$, $y(0) = 1$, $y'(0) = 0$ (15%)

(3) (a) Please obtain the determinant of the following matrix (5%)

$$\begin{bmatrix} 0 & 4 & -1 & 5 \\ -4 & 0 & 3 & -2 \\ 1 & -3 & 0 & 1 \\ -5 & 2 & -1 & 0 \end{bmatrix}$$

(b) Please list at least two properties of a singular matrix (5%)

(4) Please integrate following functions with their paths

(a) $f(z) = \frac{z^4}{z-2i}$, C is any closed path enclosing 2i. (5%)

(b) $f(z) = \frac{z\sin(3z)}{(z+4)^3}$, C is the circle $|z - 2i| = 9$. (5%)

(c) $f(z) = \frac{\ln(z-1)}{z-6}$, C is the circle $|z - 6| = 4$. (5%)

(5) Please solve the following two-dimension wave equation (35%)

$$\frac{\partial^2 u}{\partial t^2} = (\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})$$

$u(x, 0, t) = u(x, 2\pi, t) = 0$ for $t > 0$, and $0 < x < 2\pi$;

$u(0, y, t) = u(2\pi, y, t)$ for $t > 0$, and $0 < y < 2\pi$;

$u(x, y, 0) = f(x, y) = 1$; for $0 < x < 2\pi$ and $0 < y < 2\pi$.

$\frac{\partial u}{\partial t}(x, y, 0) = 0$ for $0 < x < 2\pi$ and $0 < y < 2\pi$.

試題隨卷繳回