國立臺灣大學 102 學年度碩士班招生考試試題

請詳細列出計算及推導過程，否則不予計分。題目前中括號 [ ] 內之數字為該題配分。

1. [10 points] Let $X_1, X_2, \ldots,$ be i.i.d. random variables and Y be a discrete random variable taking positive integer values. Assume that X_i's and$Y$ are independent. Let $Z = \sum_{i=1}^{Y} X_i$,

   (a) Show that $E(Z) = E(Y) \cdot E(X_1)$.

   (b) Show that $Var(Z) = E(Y) \cdot Var(X_1) + Var(Y) \cdot [E(X_1)]^2$.

2. [10 points] Let $X_1, \ldots, X_n$ be i.i.d. from the Poisson distribution $P(x|\theta) = \frac{e^{-\theta}\theta^x}{x!}$ with $\theta > 0$. Please find the UMVUE (uniformly minimum variance unbiased estimator) of $e^{-t\theta}$ with a fixed $t > 0$.

3. [30 points] Let $X_1, \ldots, X_n$ be i.i.d. random variables having the Lebesgue p.d.f. $f(x|\alpha, \beta) = \alpha\beta^{-\alpha}x^{\alpha-1}$, $0 < x < \beta$, where $\alpha > 0$ and $\beta > 0$ are unknown.

   (a) Obtain moment estimators of α and β using the method of moments.

   (b) Find the MLEs of α and β.

4. [24 points] Let $X_1, \ldots, X_m$ and $Y_1, \ldots, Y_n$ be independent random samples from Poisson distributions with mean θ_1 and θ_2, respectively. Find the likelihood ratio, Wald and score statistics for testing $H_0 : \theta_1 = k\theta_2$ versus $H_0 : \theta_1 \neq k\theta_2$, where k is a known positive constant.

5. Let $Y_1, \ldots, Y_m$ are independent $N(\mu, \sigma^2)$, while $Y_{m+1}, \ldots, Y_n$ are independent $N(\beta_0 + \beta_1 x_i, \sigma^2)$ $(i = m+1, \ldots, n)$ random samples. Note that x_i's are known constants.

   (a) [16 points] Find the MLEs for the four unknown parameters $(\mu, \beta_0, \beta_1, \sigma^2)$.

   (b) [10 points] Write our the rejection region for an approximate level α test for $H_0 : \beta_1 = 0$ versus $H_1 : \beta_1 \neq 0$. Explain how to use the test to obtain an approximate level $1 - \alpha$ confidence set for β_1.

試題隨卷繳回