共五題，總分100分

1. Please solve the following homogeneous ODE. (15%) (Hint: Frobenius method)
$4x^2y'' + 2xy' - xy = 0$

2. Solve the initial problem of the following nonhomogeneous ODE. (15%)
$y'' + 16y = 8x^2 + 64x + 17,\ y(0) = \frac{-1}{3},\ y'(0) = \frac{70}{13}$

3. (a) Please solve the linear ODE
$y'' + 8y' + 15y = \delta(t),\ y(0) = 0, y'(0) = 0$ (1)
where $\delta(t)$ is the Dirac's delta function. (10%)
(b) Based upon the solution of Eq. (1), please derive the solution of the ODE $y'' + 8y' + 15y = e^{-t}$ with the same initial conditions in (a) by convolution theorem. (10%)

4. Please integrate following functions around C counterclockwise or as indicated.
(a) $f(z) = z^{-2}\tan \pi z$, C any contour enclosing 0. (5%)
(b) $f(z) = (z-2)^{-2} Ln(z),\ C: |z-3| = 2$. (5%)
(c) $f(z) = \frac{2z^3 - 3}{z(z-1-i)^2}$, C consists of $|z| = 2$ (counterclockwise) and $|z| = 1$ clockwise. (5%)
(d) $f(z) = \frac{(1+z)\sin z}{(2z-1)^2},\ C: |z-i| = 2$. (5%)

5. Please solve the following boundary value problem using separation of variables. (30%)

$$\frac{\partial u}{\partial t} = k^2 \frac{\partial^2 u}{\partial x^2} \text{ for } 0 < x < L,\ t > 0$$
$$u(0,t) = 0,\ u(L,t) = 0, \text{ for } t \geq 0$$
$$u(x,0) = \begin{cases} x & \text{if } 0 < x < L/2 \\ L - x & \text{if } L/2 < x < L \end{cases}$$

試題隨卷繳回