

請依照題號順序作答。所有數字必須化為最簡分數或根式，未依規定者該題不予計分。

一. 填充題 (請將空格編號順序 (A), (B), ..., (I) 以及答案依順序填寫於答案

卷上。每題8分，合計72分。)

1. $\lim_{x \rightarrow \infty} x e^{-x^2/2} \int_x^{\infty} e^{-t^2/2} dt = \underline{(A)}$.

2. When $T = 30n$ and $\lim_{i \rightarrow \infty} nr_i = \lambda$, then $\lim_{i \rightarrow \infty} \prod_{i=1}^n (1+r_i) = \underline{(B)}$.

3. When $z = f(x, y)$, $f(1, y) = -y^2$ and $\frac{\partial z}{\partial x} = \ln x + 1 + 2x \ln y$, $f(x, y) = \underline{(C)}$.

4. Evaluate $I = \iint_D x^2 y^2 dx dy$, where D is the region enclosed by the curves

$y = \sqrt{x}$, $y = \sqrt{3x}$, $y = 1/x$, and $y = 2/x$. Then $I = \underline{(D)}$.

5. Let $f: R \rightarrow R$, and $z = f(xy)$. Then $x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = \underline{(E)}$.

6. The radius of convergence of the series $\sum_{i=1}^{\infty} \frac{(nx)^n}{n!}$ is $\underline{(F)}$.

7. If the ellipse $x^2/a^2 + y^2/b^2 = 1$ is to enclose the circle $x^2 + y^2 = 2y$, the values of a

and b minimize the area of the ellipse must satisfy $ab = \underline{(G)}$.

8. The value of a mortgage is defined by the recurrence relation

$$D_k = D_{k-1}e^{rh} - a(h), \quad k = 1, 2, \dots, N = T/h,$$

where $D_0 = 1$, r is a positive constant called the interest rate, T is a positive constant

called the expiry time, and $a(h)$ is the regular repayment amount. It can be proved

that $D_k = e^{rkh} - a(h) \left(\frac{e^{rkh} - 1}{e^{rh} - 1} \right)$. Since $D_N = 0$, it implies that $a(h) = \underline{(H)}$. The

total amount paid is given by $P(h) = Na(h)$. Then $\lim_{h \rightarrow 0} P(h) = \underline{(I)}$.

二. 計算題2題，每題14分，合計28分。

(a) Find and classify the stationary points of $f(x, y) = xy^2 - x^2y^2 + x^4 + 3$.

(b) Define the sequence $\{a_n\}_{n=1}^{\infty}$ recursively by $a_0 = 1$, $a_{n+1} = \sqrt{a_n + 1}$. Show that (b1)

$a_n \leq (1 + \sqrt{5})/2$; (b2) $a_{n+1} \leq a_n$, for all n ; and (b3) $\lim_{n \rightarrow \infty} a_n = (1 + \sqrt{5})/2$.

試題隨卷繳回