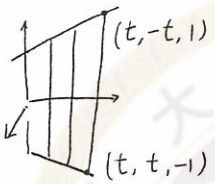
國立臺灣大學100學年度碩士班招生考試試題

1. Surface $P = \{(x, y, z) \mid z = x^2 - y^2\}$ is a hyperbolic paraboloid. $H = \{x^2 - y^2 = 1 = z\}$ is a hyperbola on P. At the point $(x, y, z) = (1, 0, 1)$, find the geodesic curvature k_g of H on P. (25/100)

2. 

(25/100) Ruled surface R is a union of straight lines joining $(t, -t, 1)$ and $(t, t, -1)$. At the point $(x, y, z) = (1, -1, 1)$, mean curvature $H = \frac{1}{2}(k_1 + k_2) = ?$

3. 2-sphere $S^2 = \{x^2 + y^2 + z^2 = 1\}$ is a surface. $\Delta \subset S^2$ is a geodesic triangle defined by $\Delta = \{x + y \geq 0,$ (25/100) $x + z \geq 0,\ x + 2y + 2z \geq 0\}$. Find the area of Δ.

4. $N = (0, 0, 1)$ is the north pole of S^2. $\Omega = S^2 - N$ is simply connected. Can you find a differential 1-form ω on Ω so that its exterior differentiation $d\omega = \sin\varphi\, d\theta \wedge d\varphi$ where θ and φ are the spherical coordinate: $x = \sin\varphi \cos\theta$, $y = \sin\varphi \sin\theta$, $z = \cos\varphi$. Can you find an η so that $d\eta = \sin\varphi\, d\theta \wedge d\varphi$ on S^2, which is simply connected too? (25/100)

試題隨卷繳回