

答案請寫於答案卷上

須列計算過程，否則不予計分

總計十題，每題十分

1. $f(x) = \begin{cases} x^2 - a & x \geq 2 \\ mx + 6 & x < 2 \end{cases}$. Find the value of m and a to make f differentiable everywhere.

2. Which of the following one(s) is(are) convergent?

(1). $\int_0^{\infty} \frac{dx}{x^3 + 1}$

(3). $\int_{-\infty}^{\infty} \sin x dx$

(2). $\int_0^2 \frac{dx}{x^2 + x - 2}$

(4). $\int_2^{\infty} \frac{dx}{x(\ln x)^p}$, $p > 1$

3. Find $(f^{-1})'(2)$, if $x^2 - (f(x))^3 = xf(x)$, $x \geq 0$.

4. Evaluate (1) $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \tan\left(\frac{i}{n}\right)$ (5 points)

(2) $\lim_{n \rightarrow \infty} \left(\frac{1}{\sqrt{2n^2 + n}} + \frac{1}{\sqrt{2n^2 + 2n}} + \frac{1}{\sqrt{2n^2 + 3n}} + \cdots + \frac{1}{\sqrt{3n^2}} \right)$ (5 points)

5. Let Ω be the solid enclosed by two solids of revolution (rotating about x -axis), one is $y = \sqrt{x}$, the other is $y = \sqrt{2-x}$. Find its surface area.

6. Find $\int_2^3 (2x+3)(x^2+3x-2)^{\frac{3}{2}} dx$.

7. Find $\frac{dy}{dx}$, given (1) $y = \frac{(x^2 - 3x - 2)^2 (2x^2 + x - 1)^3}{(x^4 - x^2 + 1)^4 (2x + 1)^6}$; (2) $y = \left[\cos(\sqrt{x} - x) \right]^3$.

(5 points each)

8. A certain region R on xy plan is bounded by three curves: $y = x^2$, $y = x^2/8$, and $y = 1/x$. Find the area of R .

9. Find the extreme value(s) of $f(x, y) = x^2 + 2xy + y^2$, given $x^2 + y^2 \leq 4$, with Lagrange multiplier.

10. Let $u = f(x, y, z) = e^{x+y-z} \sin(x^2 y + yz^2)$, $x = r \cos \theta$, $y = r \sin \theta$, $z = \theta^2$. Find $\frac{\partial u}{\partial r}$.

試題隨卷繳回