

※各大題請依題號順序作答，並請將答案填寫於答案卷內。

1. (10分) Let X be a Bernoulli random variable that takes the value 1 with probability p and the value 0 with probability $1-p$. Then the expected value of X^2 is (1a), and the variance of X is (1b).
2. (10分) Suppose X is a random vector normally distributed as $N(\mu, \Sigma)$, where μ is the mean vector and Σ is the variance-covariance matrix. Then $(X - \mu)^T \Sigma^{-1}(X - \mu)$ is distributed as (2) (where the superscript T denotes transpose and Σ^{-1} denotes the inverse matrix of Σ .)
3. (10分) Suppose a hen lays X eggs, where X is a random variable that has the Poisson distribution with parameter λ . Each egg hatches with probability p independently of the other eggs. Let K be the number of chicks. Then the conditional expectation $E(X | K)$ is (3).
4. (10分) Let X and Y be independent exponential random variables with common parameter λ . For example, the probability density function of X is $f(x) = \lambda \exp(-\lambda x)$, where $x > 0$ and $\lambda > 0$. Define $U = X + Y$ and $V = X - Y$. Then the joint probability density function of U and V is (4).
5. (10分) Suppose X and Y are independent Poisson random variables with parameters λ and ω , respectively. Define $Z = X + Y$. Then the probability of the event $\{Z = 2\}$ is (5).
6. 太平洋飲料公司開發了四種版本的新廣告，為了瞭解這四種廣告版本的優劣，該公司請來四名行銷專家加以評定，評分方式為 0 到 100 分，分數愈高表示評價愈高。四名行銷專家的給分如下，請以 $\alpha=0.05$ 檢定四種廣告版本的好壞是否一致。(15分)

評定者	廣告版本			
	版本 A	版本 B	版本 C	版本 D
專家 1	88	81	75	76
專家 2	84	79	74	68
專家 3	80	78	79	72
專家 4	79	82	81	75

7. 一項研究的目的是比較辛辛那提和匹茲堡的套房租金是否有差異。辛辛那提 35 個隨機樣本顯示平均租金為 \$370、標準差為 \$30；匹茲堡 40 個隨機樣本顯示平均租金為 \$380、標準差為 \$26。請在 0.05 的顯著水準下，檢定兩地套房租金是否有差異。
 - (a) 請計算辛辛那提套房租金的 95% 信賴區間。(5分)
 - (b) 請在 0.05 的顯著水準下，檢定匹茲堡套房租金是否大於 \$370。(5分)
 - (c) 請在 0.05 的顯著水準下，檢定兩地套房租金是否有差異。(5分)

見背面

8. A major source of revenue for any professional sports team is through ticket sales, especially sales to season ticket subscribers. A study performed a regression analysis to determine what factors caused ticket prices to vary among teams in the NBA league within a given year. The regression equation was:

$$LTIX = \alpha_0 + \alpha_1(HWIN) + \alpha_2(INCOME) + \alpha_3(PAY) + \alpha_4(POPL) + \alpha_5(TREND) + \alpha_6(CAP) + \alpha_7(STAD) + \epsilon$$

where

LTIX	= natural log of average ticket price
TIX	= average ticket price
HWIN	= average number of wins by the team in the previous three seasons
INCOME	= average income level of city population
PAY	= team payroll
POPL	= population size of city
TREND	= trends in the industry
CAP	= attendance as a percentage of capacity
STAD	= if the team is playing in a new stadium

The research gathered data covering a span of seven years (1996-2002). The results of the regression analysis can be seen in the accompanying table.

Variable	Coefficients	t Statistic	p Values
Constant	2.965	20.749	0.000
POPL	0.001	5.036	0.000
INCOME	0.003	0.208	0.836
STAD	0.108	3.180	0.002
HWIN	0.004	3.459	0.001
CAP	0.010	2.968	0.003
PAY	0.008	5.341	0.000
TREND	0.016	1.616	0.100
Adjusted R-squared		0.488	
F Statistic		28.227	
F Significance		0.000	

- (a) 請寫出這個研究者所得到的迴歸模型。(5分)
- (b) 該迴歸模型的配適度好嗎？從哪些指標得知？(3分)
- (c) 整體的迴歸模式有多少的變異解釋力？(2分)
- (d) 請檢定STAD與LTIX的線性關係，須寫出虛無假說與對立假說，檢定統計量、檢定法則與檢定結果。(5分)
- (e) 請問若球隊在新的運動場出賽，對平均票價的影響為何？(5分)

標準常態分配表

Table entry for z is the area under the standard normal curve to the left of z .

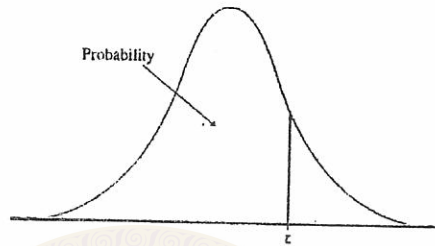


TABLE A Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

見背面

附表 Friedman 檢定
k=3

Z_r^2	$n=2$		$n=3$		$n=4$		$n=5$	
	P	Z_r^2	P	Z_r^2	P	Z_r^2	P	Z_r^2
0	1.000	.000	1.000	.0	1.000	.0	1.000	.0
1	.653	.667	.994	.5	.931	.4	.961	.4
2	.500	2.000	.528	1.5	.685	1.2	.691	.691
3	.187	4.667	.194	3.5	.273	2.8	.357	.357
4		6.000	.033	4.5	.125	3.6	.182	.182
				5.0	.069	4.8	.124	.124
				5.5	.042	5.2	.093	.093
				6.0	.028	5.4	.069	.069
				6.5	.016	5.6	.048	.048
				7.0	.008	5.8	.032	.032
				7.5	.004	6.0	.022	.022
				8.0	.002	6.2	.015	.015
				8.5	.001	6.4	.010	.010
				9.0	.000	6.6	.006	.006
				9.5	.000	6.8	.004	.004
				10.0	.000	7.0	.003	.003
				10.5	.000	7.2	.002	.002
				11.0	.000	7.4	.001	.001
				11.5	.000	7.6	.001	.001
				12.0	.000	7.8	.000	.000

附表 (續)

k=4

Z_r^2	$n=2$		$n=3$		$n=4$	
	P	Z_r^2	P	Z_r^2	P	Z_r^2
.0	1.000	.2	1.000	.0	1.000	5.7
.6	.958	.6	.958	.3	.892	6.0
1.2	.834	1.0	.810	.6	.828	6.3
1.8	.792	1.8	.727	.9	.800	6.6
2.4	.625	2.2	.608	1.2	.800	6.9
3.0	.542	2.6	.524	1.5	.754	7.2
3.6	.458	3.4	.446	1.8	.677	7.5
4.2	.375	3.8	.342	2.1	.649	7.8
4.8	.208	4.2	.200	2.4	.524	8.1
5.4	.167	5.0	.167	2.7	.508	8.4
6.0	.042	5.4	.175	3.0	.432	8.7
		6.6	.148	3.3	.389	9.0
		7.0	.084	3.6	.355	9.3
		7.4	.055	4.5	.242	10.2
		8.2	.017	4.8	.200	10.8
		9.0	.0017	5.1	.180	11.1
				5.4	.158	12.0

附表 Friedman 檢定
k=3

Z_r^2	$n=6$		$n=7$		$n=8$		$n=9$	
	P	Z_r^2	P	Z_r^2	P	Z_r^2	P	Z_r^2
.00	1.000	.000	1.000	.00	1.000	.000	1.000	.000
.33	.956	.286	.964	.25	.967	.222	.971	.222
1.00	.740	.857	.768	.75	.794	.667	.814	.667
1.33	.570	1.143	.620	1.00	.654	.889	.665	.889
2.00	.430	2.000	.486	1.75	.531	1.556	.569	1.556
2.67	.305	2.571	.305	2.25	.355	2.000	.598	2.000
3.00	.252	3.429	.257	3.00	.235	2.667	.528	2.667
4.00	.184	4.286	.192	3.25	.236	2.889	.278	2.889
4.33	.142	5.143	.142	4.00	.149	3.556	.187	3.556
5.33	.072	6.000	.085	4.75	.120	4.222	.154	4.222
6.33	.052	6.857	.052	5.25	.079	4.667	.107	4.667
7.00	.029	7.714	.027	6.00	.047	5.556	.089	5.556
8.33	.012	8.571	.021	6.75	.038	6.000	.057	6.000
9.33	.0055	9.429	.016	7.00	.030	6.222	.048	6.222
10.33	.0017	10.286	.0084	7.75	.018	6.889	.031	6.889
12.00	.00015	11.143	.0036	8.00	.0099	7.556	.019	7.556
		12.000	.0027	8.25	.0080	8.222	.016	8.222
			.0012	9.00	.0048	8.667	.010	8.667
			.0032	10.75	.0024	9.556	.0060	9.556
			.00023	12.00	.0011	10.667	.0035	10.667
			.00086	12.25	.00086	10.889	.0029	10.889
			.00026	13.00	.00026	11.556	.0013	11.556
			.00064	13.25	.00064	12.667	.00066	12.667
			.00055	14.00	.00055	13.556	.00055	13.556
			.000056	14.25	.000056	14.000	.00020	14.000
				14.50	.000097	14.222	.000097	14.222
				14.75	.000054	14.889	.000054	14.889
				15.00	.000011	16.222	.000011	16.222
				15.25	.000006	18.000	.000006	18.000

試題隨卷繳回