

1. Find the symmetric factorization $A = LDL^T$ of $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 6 & 4 \\ 0 & 4 & 11 \end{bmatrix}$. (10%)
2. True or false, with reason if true or counterexample if false: (20%)
 - (a) If A and B are symmetric then AB is symmetric.
 - (b) If $A^2 + A = I$ then $A^{-1} = A + I$.
 - (c) If $A^T = -A$ then the row space of A equals the column space.
 - (d) If V is orthogonal to W , then V^\perp is orthogonal to W^\perp .
 - (e) If A is invertible and B is singular, then $A + B$ is invertible.
3. Find a best approximation to $y = x^3$ by a straight line between $x = 0$ and $x = 1$. (20%)
4. If P_1 is an even permutation matrix and P_2 is odd, deduce from $P_1 + P_2 = P_1(P_1^T + P_2^T)P_2$ that $\det(P_1 + P_2) = 0$. (10%)
5. Find the general solution of the equation $\frac{d^2u}{dt^2} = \begin{bmatrix} -5 & 4 \\ 4 & -5 \end{bmatrix} u$. (20%)
6. Decide whether $F = y/x^2 + x/y^2 + xy$ has a minimum at the point $(1, 1)$ with reason. (20%)

試題隨卷繳回