

1. (15%) Three sinusoidally time-varying, linearly polarized vector fields are given at a point by

$$\vec{E}_1 = \sqrt{3}\vec{a}_x \cos(10^9 t + 30^\circ)$$

$$\vec{E}_2 = \vec{a}_z \cos(10^9 t + 30^\circ)$$

$$\vec{E}_3 = \sqrt{3}\vec{a}_y \cos(10^9 t - 60^\circ)$$

(a) Find the polarization of the vector field $\vec{E}_1 + \vec{E}_2$. (7%)

(b) Find the polarization of the vector field $\vec{E}_1 + \vec{E}_3$. (8%)

2. (15%) Given that $\vec{H} = H_1(t - az)\vec{a}_y$ and $\vec{D} = aH_1(t - az)\vec{a}_x$ for $z > 0$ and $\vec{H} = -H_1(t + az)\vec{a}_y$ and $\vec{D} = aH_1(t + az)\vec{a}_x$ for $z < 0$, find the current due to flow of charges enclosed by the rectangular closed path from $(0, 0, 1)$ to $(0, 1, 1)$ to $(0, 1, -1)$ to $(0, 0, -1)$ to $(0, 0, 1)$, where H_1 and a are constants.

3. (15%) The region $x < 0$ is a free space and the region $x > 0$ is a perfect dielectric medium. At a point on the boundary, the electric field on the $-x$ side is $\vec{E}_1 = E_0(4\vec{a}_x + 2\vec{a}_y + 5\vec{a}_z)$, whereas on the $+x$ side, it is $\vec{E}_2 = 3E_0(\vec{a}_x + \vec{a}_z)$, where E_0 is a constant. Find the permittivity of the dielectric medium.

4. (25%) The electric field of a uniform plane wave propagating in free space is given by

$$\vec{E} = 100 \cos(3 \times 10^9 t + 10y) \vec{a}_x \text{ V/m}$$

(a) Find the direction of propagation of the wave. (5%)

(b) Find the frequency. (5%)

(c) Find the wavelength. (5%)

(d) Find the associated magnetic field \vec{H} . (5%)

(e) Find the instantaneous power flow across a surface of area 1 m^2 in the $y=0$ plane at $t=0$. (5%)

5. (15%) Charge is distributed with density $\rho(x, y, z) = \rho_0(10 - x^2 - y^2 - z^2)$ C/m^3 in a cubic box bounded by the planes $x = \pm 1 \text{ m}$, $y = \pm 1 \text{ m}$, and $z = \pm 1 \text{ m}$. Find the displacement flux emanating from the surface of the box.

6. (a) State Stokes' theorem. (7%)

(b) State Ampère's circuital law in differential and integral forms. (8%)