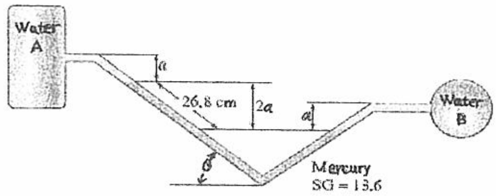
(1) (20) (a) (15) Prove that curves of constant ψ (stream function) are streamlines of the flow. (b) (5) Explain the physical meaning of the difference between two streamlines.

(2) (20) Two water tanks are connected to each other through a mercury manometer with inclined tubes. If the pressure difference between the two tanks is 20 kPa, calculate a and θ. (The density of water is 1,000 kg/m^3)



(3) (20) Oil at 20°C (ρ = 888 kg/m^3 and μ = 0.800 kg/m · s) is flowing steadily through a 5-cm-diameter 40-m-long horizontal pipe. The flow is fully developed and the minor loss at the inlet is negligible. The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe. Also verify that the flow through the pipe is laminar.

(4) (20) Water enters a four-armed lawn sprinkler along the vertical axis at a rate of 120 L/s, and leaves the sprinkler nozzles as 2-cm diameter jets in the tangential direction. The length of each sprinkler arm is 0.45 m. Disregarding any frictional effects, determine the rate of rotation of the sprinkler in rev/min.

(5) (20) Simplify the Navier–Stokes equation as much as possible for the case of incompressible static flow, with gravity acting in the negative z-direction. Begin with the incompressible vector form of the Navier–Stokes equation, explain how and why some terms can be simplified, and give your final result as a vector equation. Given: $\rho \frac{D\vec{V}}{Dt} = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}$