國立臺灣大學99學年度碩士班招生考試試題

1. (15%) Let $X_1, \cdots, X_n$ be a random sample from a population with finite moments $\mu_j = E[X_1^j]$, $j = 1, 2, 3, 4$, and S_n^2 be the usual unbiased sample variance. Compute the mean squared error of S_n^2.

2. (10%) Let X and Y be mutually independent continuous random variables with the distributions $F_X(x)$ and $F_Y(y)$, respectively. Derive the distribution of X conditioning on $\{Z = 0\}$, where $Z = I(X \leq Y)$.

3. (10%) Let X be a random variable with the cumulative distribution function $F(x)$. Show that $P(F(X) > u) \geq (1 - u)$ for $u \in (0, 1)$.

4. (10%) Specify the joint distribution of R and Θ so that $X = R\cos\Theta$ and $Y = R\sin\Theta$ are independent standard normal random variables.

5. (15%) Let $X_1, \cdots, X_n, X_{n+1}$ be a random sample from a uniform distribution $U(0, 1)$, and $X_{(k)}$ and $X_{(m)}$ be the kth and the mth order statistics of $\{X_n, \cdots, X_n\}$, $1 < k < m < n$. Compute the probability $P(X_{(k)} < X_{n+1} < X_{(m)})$.

6. (15%) Let $X_1, \cdots, X_{n+1}$ be a random sample from $Bernoulli(\pi)$ and $h(\pi) = P(\sum_{i=1}^{n} X_i > X_{n+1}|\pi)$. Find the uniformly minimum variance unbiased estimator of $h(\pi)$.

7. (15%) Let $X_1, \cdots, X_n$ be a random sample from a Poisson distribution with rate λ. Derive the uniformly most powerful level α test, $0 < \alpha < 1$, for the hypotheses $H_0 : \lambda = \lambda_0$ versus $H_A : \lambda > \lambda_0$.

8 (10%). Let $X_1, \cdots, X_n$ be a random sample from a normal distribution with mean θ and a known variance σ^2, and θ have a prior normal distribution with known mean μ and variance τ^2. Find the Bayes estimator of θ based on the loss function $L(\theta, \delta(X_1, \cdots, X_n)) = |\delta(X_1, \cdots, X_n) - \theta|$