

(1)-(10)題各 8 分，按題序標清題號寫下答案，其他計算式一律不予計分。

1. A car is racing along the elliptical route  $x^2/9 + y^2/16 = 2500m^2$ . When the car is at the point (120m, 120m), a man at the point (30m, 0) determines with radar that the distance between him and the car is increasing at the rate 37m/sec. Find the speed (m/sec) of the car at that moment.

2. Find  $\lim_{x \rightarrow 0} \left( \frac{x}{\sin^3 x} - \frac{1}{x^2} \right)$ .

3. Given  $f(x) = \ln(1 + x + x^2)$ , find  $f^{(3k)}(0)$ , where  $k \geq 0$  is an integer.

4. Find the arc length of the curve  $y = x^2/2$  from  $x = 0$  to  $x = 1$ .

5. Find the volume of the solid generated by revolving the region enclosed by the curve  $y = \frac{2x}{4 - x^2}$ , the  $x$ -axis,  $x = 0$  and  $x = 1$ .

6. Let  $g(x)$  be the inverse function of  $f(x) = x(\ln x)^3$  for  $x \geq 1$ . Find  $\int_0^e g(x) dx$ .

7. Find the tangent plane to the surface  $x^3 + y^3 + z^3 - 3xy^2z^3 = 0$  at the point  $x = 1, y = 1$  and  $z = 1$ .

8. Find the local minimum values of  $f(x, y) = x^3 - 12xy + 8y^3$ .

9. Evaluate  $\int_{y=0}^{y=2} \int_{x=y}^{x=2} e^{x^2} dx dy$ .

10. Solve the differential equation  $y' \cos x + y \sin x = \cos^3 x, y(\pi/4) = 1$ .

(A)、(B)兩題各 10 分，請寫出詳盡之計算與論證過程。

A. Evaluate  $\iint_{\Omega} \sqrt{x^2 + xy + y^2} dx dy$ , where  $\Omega = \{(x, y) : x^2 + xy + y^2 \leq 1\}$ , by first making the change of variables  $u = x + \frac{y}{2}, v = \frac{\sqrt{3}y}{2}$  and then by polar coordinates.

B. Let  $\Gamma$  be the intersection curve of the two surfaces  $x^3 + 2y^3 + 3z^3 - 2xyz = 4$  and  $x^2 + y^2 + z^2 = 3$ . Find the unit tangent vector to  $\Gamma$  at the point (1, 1, 1)

試題隨卷繳回