

1.(20 points) Suppose the Neumann problem

$$\begin{aligned}\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} &= 0, & 0 < x < 1, 0 < y < 1, \\ \frac{\partial u}{\partial y}(x, 0) &= 0, \frac{\partial u}{\partial y}(x, 1) = 0, & 0 < x < 1, \\ \frac{\partial u}{\partial x}(0, y) &= 0, \frac{\partial u}{\partial x}(1, y) = g(y), & 0 < y < 1\end{aligned}$$

has a smooth solution. Find $\int_0^1 g(y) dy$.

2.(20 points) Solve the partial differential equation

$$\begin{cases} u_x + u_y + 2u = \cos y, \\ u(x, 0) = 0. \end{cases}$$

3.(20 points) Find the solution of the initial value problem

$$x'(t) = x(t)(1 - x(t)), \quad x(0) = 0.01,$$

and calculate $\lim_{t \rightarrow \infty} x(t)$.

4.(20 points) Find the general solution of the differential equation

$$x''(t) = 2t(x'(t))^2.$$

5.(20 points) Find the solution of the differential equation

$$y'' - y = \cos 2t,$$

satisfying the initial condition

$$y(0) = 0, \quad y'(0) = 1.$$