國立臺灣大學 112 學年度碩士班招生考試試題

Any device with computer algebra system is prohibited during the exam. Solve the following problems and explain your reasoning.

1. Suppose that $f(x)$ is differentiable and
$$\lim_{x\to 0}\frac{f(x)-e^x-2x}{x^2}=3.$$
   (a) (4 pts) Write down the linearization of $f(x)$ at $x=0$.
   (b) (5 pts) Suppose that $f''(0)$ exists. Find $f''(0)$.
   (c) (5 pts)(Continued) Suppose that near $(0,0)$, y is a function of x defined implicitly by the equation $f(2y)+\sin x=f(0)$. Compute $\frac{d^2y}{dx^2}$ at $(0,0)$.

2. Consider $f(x)=\dfrac{(\ln|x|)^2}{x}$.
   (a) (4 pts) Find horizontal and vertical asymptotes of $y=f(x)$.
   (b) (6 pts) Compute $f'(x)$. Find intervals of increase and intervals of decrease of $f(x)$.
   (c) (6 pts) Copute $f''(x)$. Discuss concavity of $y=f(x)$.
   (d) (4 pts) Sketch the curve $y=f(x)$.

3. (8 pts) Suppose that $f(t)$ is a continuous function and
$$y(x)=\int_0^x f(t)\sin(x-t)\,dt.$$
Compute $y''(x)+y(x)$.

4. (a) (12 pts) Compute the integral $\displaystyle\int\frac{x^2}{(4x^2+1)^3}dx$.
   (b) (12 pts) Compute the integral $\displaystyle\iiint_S\frac{dV}{x^2+y^2+(2-z)^2}$ where $S=\{(x,y,z)\mid \frac{1}{4}\le x^2+y^2+z^2\le 1 \text{ and } z\ge 0\}$.

5. (10 pts) Find the maximum and minimum values of x^2+yz+z^2 on the ball $x^2+y^2+z^2\le 1$.

6. (12 pts) Find the gravitational attraction of a circular cylindrical shell of radius a, height h, and a constant areal density σ on a particle of mass m located on the axis of the cylinder b units above the base.

7. (12 pts) Compute $\displaystyle\iint_S\vec{F}(x,y,z)\cdot d\vec{S}$ where $\vec{F}(x,y,z)=\dfrac{x\vec{i}+y\vec{j}+z\vec{k}}{(x^2+y^2+z^2)^{3/2}}$ and S is the part of the plane $z=2$ inside the cone $z=\sqrt{3(x^2+y^2+z^2)}$ with upward orientation.

試題隨卷繳回