

Please write down all the critical steps in your answer. You can answer your questions in English or in Chinese.
(請寫出重要計算步驟。可以用英文或中文作答)

Problem 1 (20 points)

Test, for dimensional homogeneity, the following formula for volume flow Q through a hole of diameter D in the side of a tank whose liquid surface is a distance h above the hole position:

$$Q = 0.68D^2\sqrt{gh}$$

where g is the acceleration of gravity. What are the dimensions of the constant 0.68?

Problem 2 (20 points)

The pipe flow in Fig. 1 fills a cylindrical tank as shown. At time $t = 0$, the water depth in the tank is 30 cm. Estimate the time required to fill the remainder of the tank.

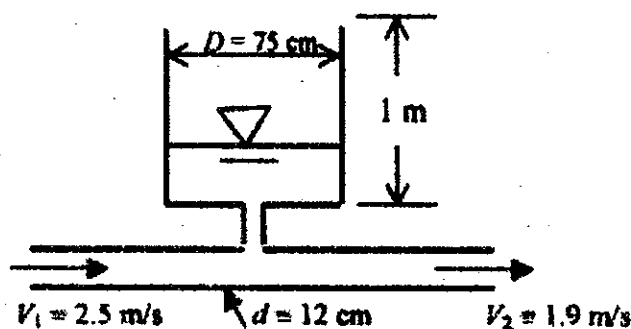


Figure 1.

Problem 3 (20 points)

A vertical lock gate is 4 m wide and separates 20°C water levels of 2 m and 3 m respectively. Find the moment about the bottom required to keep the gate stationary.

Problem 4 (20 points)

A two-dimensional velocity field is given by

$$V = (x^2 - y^2 + x)i - (2xy + y)j$$

in arbitrary units. At $(x, y) = (1, 2)$, compute (a) the accelerations a_x and a_y , and (b) the velocity component in the direction $\theta = 40^\circ$.

Problem 5 (20 points)

Consider a steady, two-dimensional, incompressible flow of a Newtonian fluid with the velocity field $u = -2xy$, $v = y^2 - x^2$, and $w = 0$. (a) Does this flow satisfy conservation of mass? (b) Find the pressure field $p(x, y)$ if the pressure at point $(x = 0, y = 0)$ is equal to P_a .