

※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. Multiple Choice Questions (50%)

1. What is the coefficient of y^3x^6 in $(1+x+y)^5(1+x)^7$?

- (A) 350
- (B) 840
- (C) 70
- (D) 490
- (E) None of the above.

2. The volume of a ball of radius R in 6-dimensional Euclidean space is $\frac{\pi^3}{6}R^6$. What is the volume of a ball of radius R in 7-dimensional Euclidean space?

- (A) $\frac{8\pi^3}{105}R^7$
- (B) $\frac{32\pi^3}{105}R^7$
- (C) $\frac{16\pi^3}{105}R^7$
- (D) $\frac{4\pi^3}{105}R^7$
- (E) None of the above.

3. The volume of a ball of radius R in 6-dimensional Euclidean space is $\frac{\pi^3}{6}R^6$. What is the surface area of this ball?

- (A) $\frac{\pi^3}{6}R^5$
- (B) $\frac{\pi^3}{3}R^5$
- (C) $\frac{\pi^3}{2}R^5$
- (D) π^3R^5
- (E) None of the above.

4. Which of the following is closest to the value of $\int_0^1 \sqrt{1 + \frac{1}{3x}} dx$?

- (A) 1
- (B) 1.2
- (C) 1.6
- (D) 2
- (E) The integral doesn't converge.

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5. $\int_{e^{-3}}^{e^{-2}} \frac{1}{x \ln x} dx =$

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\ln \frac{2}{3}$

(D) $\ln \frac{3}{2}$

(E) None of the above.

6. How many roots of $x^4 - 4x^2 - 8x + 12$ lie in the range $[-2, 2]$?

(A) 0

(B) 1

(C) 2

(D) 3

(E) None of the above.

7. Assume $f: \mathbb{R} \rightarrow \mathbb{R}$ is smooth. $\lim_{h \rightarrow 0} \frac{f(x+4h) - 2f(x) + f(x-4h)}{h^2} =$

(A) 0

(B) $8f'(x)$

(C) $8f''(x)$

(D) $16f''(x)$

(E) None of the above.

8. From the following list, choose the smallest value of n for which the following limit exists for all $r \geq n$.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^r}{|x|^2 + |y|^2}$$

(A) 1

(B) 1.5

(C) 2

(D) 2.5

(E) None of the above.

9. Find the maximum of x^2y on the curve $x^2 + 2y^2 = 6$.

(A) 3

(B) 4

(C) 5

(D) 6

(E) None of the above.

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10. Assume $f: R \rightarrow R$ is smooth with $f(1) = 1$ and $f'(1) = 2$. Find $\frac{d}{dx} \left(\frac{f(e^{2x-2})}{xf(x)} \right)$ at $x=1$.

- (A) 0
- (B) $e^2 - 3$
- (C) $e - 3$
- (D) 1
- (E) None of the above.

※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

2. Answer the following questions:

1. $\lim_{x \rightarrow \infty} \frac{(2\pi)^{\frac{1}{2}} x^{x-\frac{1}{2}} e^{-x}}{\int_0^{\infty} y^{x-1} e^{-y} dy} = \underline{\hspace{2cm}}$ (10%)

2. $\int_0^1 \frac{41!}{x!(40-x)!} y^{x+2} (1-y)^{40-x} dy = \underline{\hspace{2cm}}$ (10%)

3. $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} (x-\mu)^{2n} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx = \underline{\hspace{2cm}}$ where $\mu > 0$ and $\sigma > 0$ (10%)

4. $\frac{\int_0^{\infty} \frac{1}{\int_0^{\infty} y^{\alpha-1} e^{-y} dy} x^{-2} (\beta x)^{\alpha} e^{-\beta x} dx}{\int_0^{\infty} y^{\alpha-1} e^{-y} dy} = \underline{\hspace{2cm}}$ where $\alpha > 1$ (10%)

5. $\int_0^1 x^{r+\alpha-1} (1-x)^{s+\beta-1} \frac{\int_0^{\infty} y^{\alpha+\beta-1} e^{-y} dy}{\int_0^{\infty} z^{\alpha-1} e^{-z} dz \times \int_0^{\infty} t^{\beta-1} e^{-t} dt} dx = \underline{\hspace{2cm}}$ (10%)