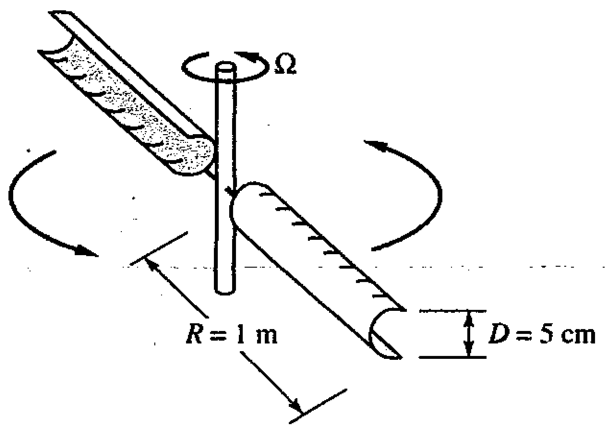
1. (25%) A rotary mixer consists of two 1-m-long half-tubes rotating around a central arm, as shown in the figure (left). Using the drag coefficient C_D from the table (right), (a) derive an expression for the torque T required to drive the mixer at angular velocity Ω in a fluid of density ρ. Suppose that the fluid is water at 20°C and the maximum driving power P available is 20 kW (b) What is the maximum rotation speed Ω_{max} (rev/min)?

Hint: 1. For water at 20°C, take ρ=998 kg/m^3 and μ=0.001 kg/(m·s).

2. Torque is equal to the integral of force over two half-tubes: $T=2\int_{\text{half-tube}} r dF$, where F is the drag force.

3. The drag coefficient is defined as $C_D=\dfrac{2F}{\rho V^2 A}$, where V is the local fluid velocity ($V=\Omega r$) and A is the frontal area ($dA=Ddr$).

4. Power is equal to the product of torque and rotation speed ($P=T\Omega$) and 1 W= 1 N·m/s.



Shape	C_D based on frontal area	Shape	C_D based on frontal area	Shape	C_D based on frontal area
Square cylinder:	2.1	Half cylinder:	1.2	Plate:	2.0
	1.6		1.7	Thin plate normal to a wall:	1.4
Half tube:	1.2	Equilateral triangle:	1.6		
	2.3		2.0	Hexagon:	1.0 ↕ 0.7

2. (25%) Air at 20°C and 1 atm enters a 40-cm-square duct as in shown in the figure (left). Using the "displacement thickness" concept of the figure (right), estimate (*a*) the mean velocity (m/s) and (*b*) the mean *gage* pressure (Pa) in the core of the flow at the position x = 3 m. (*c*) What is the average pressure gradient (Pa/m) in this section?

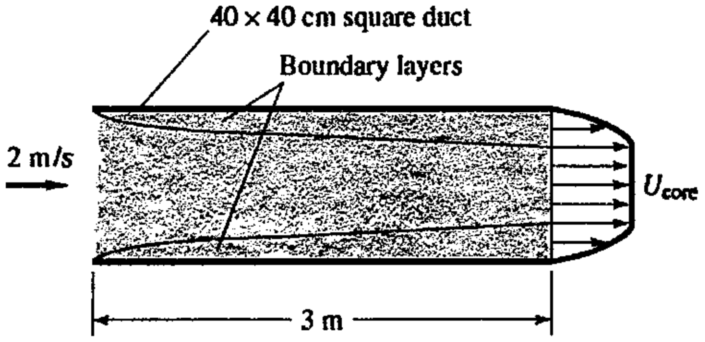
Hint: 1. For air at 20°C, take ρ=1.2 kg/m^3 and μ=1.8 ×10^{-5} kg/(m·s).

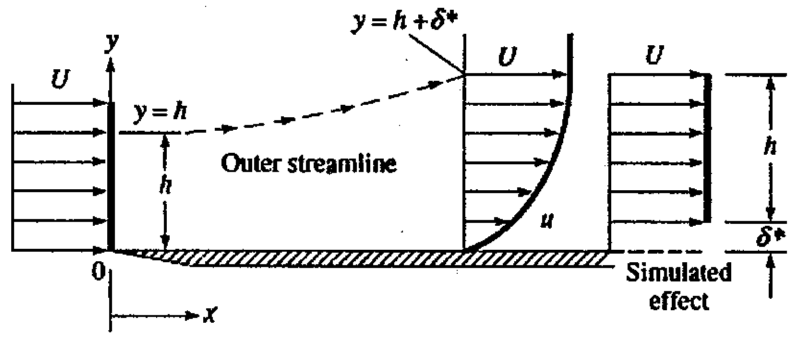
2. The mean velocity at the exit is estimated by the continuity of flow, that is, Q(entrance)= Q(exit), where Q is the volume flow rate ($Q=VA$, V is the mean velocity and A is the effective cross sectional area).

3. The pressure change in the (frictionless) core flow is estimated from Bernoulli's equation:

$$p+\frac{1}{2}\rho V^2\Big|_{\text{entrance}}=p+\frac{1}{2}\rho V^2\Big|_{\text{exit}}$$, where p is the mean pressure.

4. Gage pressure is equal to absolute pressure minus atmospheric pressure: p(gage)=$p-p_{atm}$ and 1 Pa=1 N/m^2.





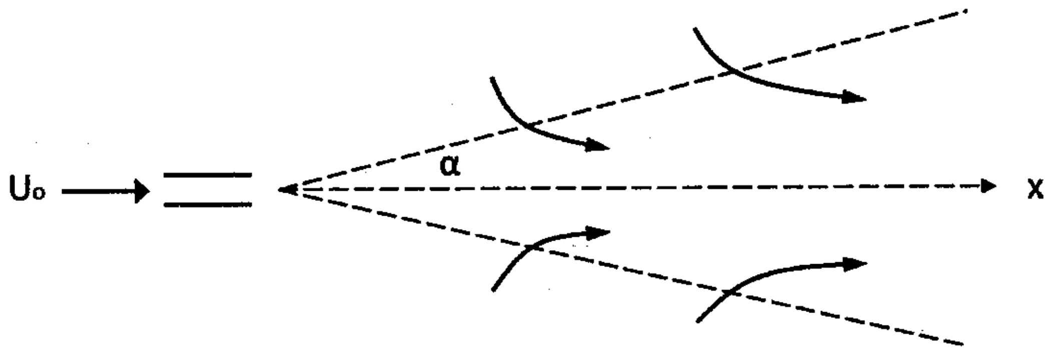
$$\frac{\delta^*}{x}=\frac{1.721}{Re_x^{1/2}}$$

3. (25%) A liquid of density ρ_l is sprayed from a nozzle into still air of density ρ_a, as shown in the figure. The liquid stream emerges from the nozzle at a speed U_0 and break up into small droplets, entraining air and accelerating it in the direction of the spray axis x. The mass flow rate of the liquid leaving the nozzle is $\dot{m}$. It is observed that the liquid droplets are uniformly distributed over a cross-section of a cone of half angle α and have the same speed U within that cross-section (i.e., both U_0 and U are uniform across the jet cross-section).

By using photography, the speed $U(x)$ of the spray droplets is measured as a function of the distance x from the tip of the nozzle.

(a) (15%) Derive an expression for the mean speed $V(x)$ of the air within the spray cone, as a function of x and the known flow variables, assuming that V is uniform across the jet cross-section.

(b) (10%) The spray is observed to consist of droplets of uniform diameter d. Derive an expression for the number of droplets per unit volume, $n(x)$, as a function of flow parameters.



4. (25%)An aerosol generator contains two tubes which are joined to form a T junction, as shown in the sketch. The larger diameter tube (diameter D) opens to a pressurized chamber at a gage pressure ΔP containing a gas of density ρ_g, say air. Its other end is open to the atmosphere. The open end of the thinner tube (diameter d) is submerged in a liquid of density ρ_l. The length of the thin tube is L.

Suppose that $\Delta P/P_{atm} \sim O(0.1)$, and that both the air and liquid flows are **inviscid**. Also assume that gravity does not affect the flow.

(a) (5%) Sketch the flow pattern that you expect, **including** the flow near the T junction.

(b) (10%) What is the ratio of the volumetric flow rate of liquid to gas $\dot{Q}_l/\dot{Q}_g$ at the exit from the atomizer?

(c) (10%) When it is **not** relevant to neglect the effect of gravity? Give your result in a non-dimensional form.