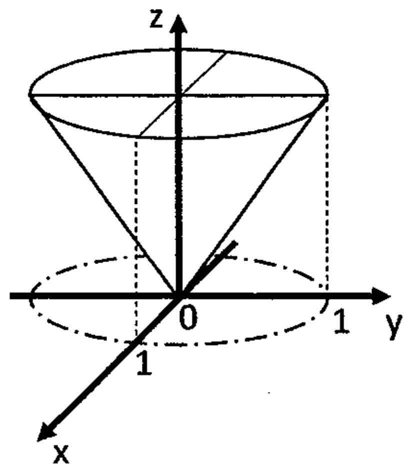
1. Use the Divergence Theorem to evaluate the surface integral $\iint_S F \cdot dS$ of the vector field $F(x,y,z) = (x^3, y^3, z^3)$, where S is the surface of a solid bounded by the cone $x^2 + y^2 - z^2 = 0$ and the plane $z = 1$. (15%)



2. Solve the differential equation $y'' + 4y' + 5y = 0$. (10%)

3. Find the third derivative of the function $y = e^{2x} \ln x$. (10%)

4. Use power series to solve the differential equation: $y'' + x^2y' + xy = 0$, $y(0) = 0, y'(0) = 1$. (15%)

5. Which of the following matrices are Hermitian ? Normal ? Why? (20%)

$$\mathbf{A} = \begin{bmatrix} 3 & 2+2i \\ 2-2i & -3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 2i & \sqrt{3} \\ \sqrt{3} & -2i \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 2i & 1 \\ 2i & 1 & -1+i \\ -1 & 1+i & 1 \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} 1 & 1+i & -i \\ 1-i & 3 & 2 \\ i & 2 & 1 \end{bmatrix}$$

6. If we consider scalar field of pollutant concentration in space given by a scalar function $f(P) = f(x,y,z)$, what is the gradient of this scalar field function? What is the divergence of this gradient? Please explain physical meanings of the gradient and divergence. (15%)

7. Find the inverse Laplace Transform of $F(s)$:

$$F(s) = p(s)/q(s) = \left(A_j s^j + A_{j-1}s^{j-1} + \cdots + A_1 s + A_0\right)/\left(s^i + B_{i-1}s^{i-1} + \cdots + B_1 s + B_0\right)$$

where $i > j$, As and Bs are real constants, and if i roots of $q(s) = 0$ are real and nonrepeated. (15%)

試題隨卷繳回