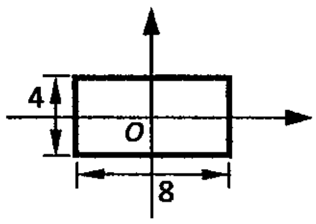
1. (5% for each question, total 50%) Answer the following questions. **(Derivations are <u>not</u> required)**

(a) The following table lists the measurements of the deformation angle θ of a fluid element with time t under the action of a constant shearing stress. Is the fluid Newtonian or non-Newtonian?

θ (°)	1.2	4.8	10.8	19.2
t (sec)	1	2	3	4

(b) A 2-D incompressible flow has velocity components in polar coordinates (r,θ): $v_r = A/r$, $v_\theta = 0$ where A is a constant. Find the net volume flow rate Q across the rectangular boundary as shown below.



(Assume unit width into the paper)

(c) If the velocity of an incompressible flow field is found to be proportional to the pressure gradient of the flow : $\underline{v} = -C\nabla p$ where C is a constant, then what is the value of $\nabla^2 p$?

(d) Following the above problem, is the flow rotational or irrotational?

(e) Let $\underline{v}$ denote the velocity field of a flow and ρ the density of the fluid. Which of the followings is the correct expression for the momentum flux across an elemental area dA (with unit normal vector $\underline{n}$) in the flow field?
(A) $\rho|\underline{v}|^2 dA$　(B) $\rho\underline{v}\cdot\underline{n}dA$　(C) $\rho|\underline{v}|(\underline{v}\cdot\underline{n})dA$　(D) $\rho\underline{v}(\underline{v}\cdot\underline{n})dA$　(E) $\rho|\underline{v}|^2\underline{n}dA$

(f) The inviscid flow distribution at the outer edge of a 2-D incompressible boundary layer has the form $U(x) = Cx^\alpha$. Find the value of α under which the pressure gradient inside the boundary-layer is a constant.

(g) The drag force F experienced by a cylinder of diameter D in a uniform stream of flow speed V and density ρ is given by :

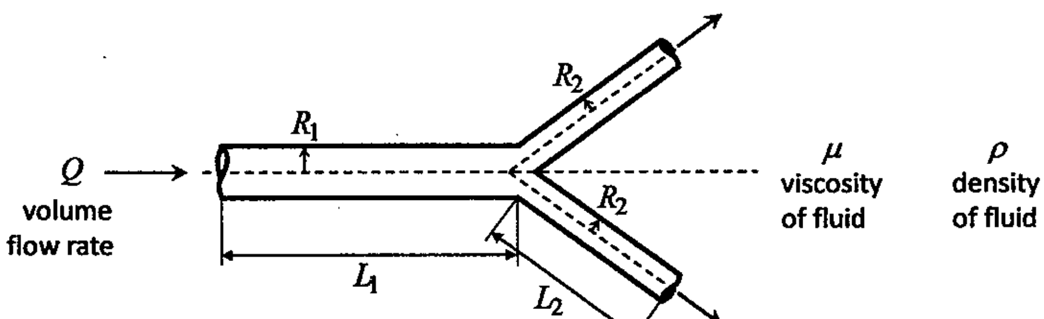
$$\frac{F}{\rho V^2 D} = f(Re) \qquad \text{where } Re \text{ is the Reynolds number.}$$

If it is observed from experiments that the drag force F is linearly proportional to the free stream speed V, then what is the general expression for the function f?

(h) How does fluid turbulence generally affect (increase or reduce) the friction drag on a solid body?

(i) A fluid of density ρ and viscosity μ flows through a pipe of diameter D and length L with an averaged flow speed V. If the pressure drop Δp between the inlet and outlet of the pipe is found to be $\Delta p \approx CL\rho^{3/4}\mu^{1/4}D^{-5/4}V^{\alpha}$ where C is a dimensionless constant, then what is the value of α?

(j) A branching circular pipe system is shown as below. Assume Poiseuille solution $\Delta p = \dfrac{8\mu QL}{\pi R^4}$ for the circular pipe flow applies to each segment of the system, what is the expression for the total friction head loss h_f of this pipe flow (in terms of the relevant parameters specified in the figure below)? Neglect all minor losses.



見背面

2. (15%) Consider a general control volume in fluids. The fluid has a density ρ and a velocity field $\underline{u}$. The control volume V_{CV} and the corresponding control surface A_{cs} are time-varying quantities. The velocity of the control surface is $\underline{U}_{cs}$ and the unit normal vector of the control surface is $\hat{n}$. Derive the control volume form of the mass conservation

$$\frac{d}{dt}\int_{V_{CV}} \rho\, dV + \oint_{A_{cs}} \rho\left(\underline{u} - \underline{U}_{cs}\right)\cdot \hat{n}\, dA = 0$$

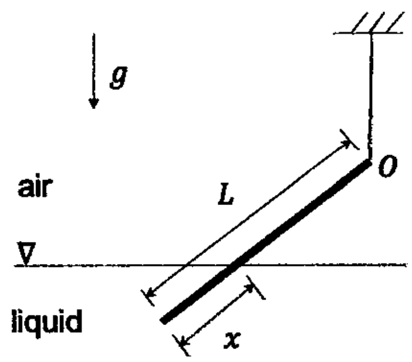
from the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\left(\rho \underline{u}\right) = 0.$$

3. (20%) As shown in the following figure, a thin cylinder bar with a density ρ_s is hanged with a string at the end O and partially submerged in a liquid fluid with a density ρ_f. The length of the bar is L and the length of the bar submerged in the fluid is x. Let $\rho_s < \rho_f$ so that the bar is floating on the liquid. The magnitude of the gravitational acceleration is g.

(a) (5%) Determine the ratio $\frac{x}{L}$ in terms of the density ratio $\frac{\rho_s}{\rho_f}$.

(b) (15%) If the bar is perturbed and oscillates about O with a frequency f, perform a dimensional analysis (8%) to determine how will the frequency f behave with respect to the bar length L at a fixed density ratio $\frac{\rho_s}{\rho_f}$ (7%).

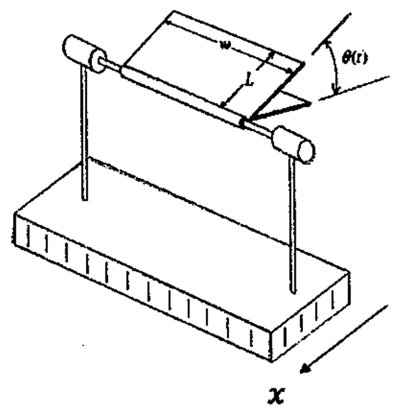


4. (15%) An inventor proposes to propel a small, low-speed underwater vehicle with a device that successively closes and opens two rigid flaps as shown above. The sketch shows a scale model being tested in a large, deep pool containing stationary water (density ρ). In one cycle, the system starts at its maximum open angle θ_1, closes at a uniform angular speed

$$\frac{d\theta}{dt} = -\omega$$

until it reaches a smaller angle θ_2, and then opens at the same angular speed ω until it returns to its original angle.

Note that, unlike the depiction in the sketch, $w \ll L$ *and* $\theta \ll 1$.



(a) (5%) Which direction will the vehicle move ($+x$ or $-x$ or not moving at all)?

(b) (5%) Make an argument of your answer in part (a).

(c) (5%) Using the dimensional and scale analyses, estimate the scale of the thrust that the vehicle generates.

試題隨卷繳回