1. Find the $PA = LDU$ factorization for $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$. (20%)

2. Find a basis for each of the four subspaces of $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

   (20%)

3. True or false, with reason if true and counterexample if false: (20%)
   (a) If $L_1U_1 = L_2U_2$ (upper triangular U's with nonzero diagonal, lower triangular L's with unit diagonal), then $L_1 = L_2$ and $U_1 = U_2$. The LU factorization is unique.
   (b) If $A^2 + A = I$ then $A^{-1} = A + I$.
   (c) If all diagonal entries of A are zero, then A is singular.
   (d) If columns 1 and 3 of B are the same, so are columns 1 and 3 of AB.

4. If $A = S\Lambda S^{-1}$, diagonalize the block matrix $B = \begin{bmatrix} A & 0 \\ 0 & 2A \end{bmatrix}$. Find its eigenvalues and eigenvector matrix. (20%)

5. Show the condition that $ax^2 + 2bxy + cy^2$ is positive definite. Decide whether $F = x^2y^2 - 2x - 2y$ has a minimum. (20%)

試題隨卷繳回