

※ 注意：請於答案卷上依序作答，並應註明作答之大題及其題號。

1. Compute the following integrals.

i) (10 %)

$$\int x^3 e^{x^2} dx.$$

ii) (10 %)

$$\int_0^{\infty} \frac{dx}{x^2 + 2x + 2}.$$

2. Let

$$f(x) = \frac{x^2}{\sqrt{1+x^2}} - x^2$$

$$g(x) = \cos x \sin(x^2) - x^2.$$

i) (10 %) Compute the coefficients of x^4 and x^9 in the Taylor series of $f(x)$ at $x = 0$.

ii) (10 %) Compute the coefficients of x^4 and x^6 in the Taylor series of $g(x)$ at $x = 0$.

iii) (10 %) Compute $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)}$.

3. Let $\Omega = \{(x, y) | x^2 + y^2 \leq 1\}$. Given

$$A = \iint_{\Omega} \sin(x^2) \cos(y^2) dx dy$$

$$B = \iint_{\Omega} \cos(x^2) \sin(y^2) dx dy.$$

Let $C = A + B$. Then C is equal to an iterated integral which is expressed in terms of polar coordinates r and θ .

i) (10 %) First derive this iterated integral expression for C in polar coordinates, then compute the value of C .

ii) (10 %) Prove that $A = B$.

4. Let $f(x, y) = x^2 + 4xy + y^2$. We call the region $\{(x, y) | x^2 + y^2 < 1\}$ open disk D_1 , $\{(x, y) | x^2 + y^2 \leq 1\}$ the closed disk D_2 , and $\{(x, y) | x^2 + y^2 = 1\}$ the circle C .

i) (10 %) On the open disk D_1 find, if there is any, those points on which $f(x, y)$ is a relative minimum, relative maximum or a saddle point.

ii) (10 %) On the circle C compute the absolute maximum value and absolute minimum value of $f(x, y)$.

iii) (10 %) On the closed disk D_2 give your reasons to find the absolute maximum value and absolute minimum value of $f(x, y)$.

試題隨卷繳回