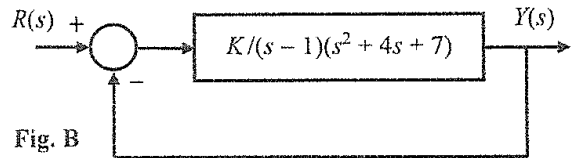
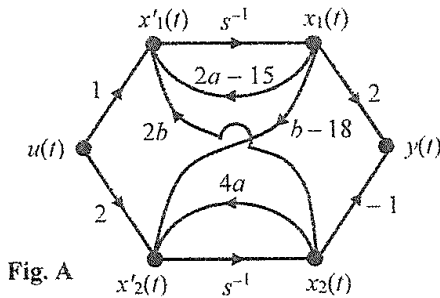


※ 注意：請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

1. A state diagram of a control system is given in Fig. A, where  $a$  and  $b$  are constants. (1) Write the dynamic equation of the system.【計分：3分】 (2) Determine the conditions on  $a$  and  $b$  so that the system is completely controllable and observable.【計分：4分】 (3) Let  $a = 4$  and  $b = 3$ , (i) find the transfer function  $Y(s)/U(s)$ 【計分：3分】 and (ii) determine the state-transition matrix  $\phi(t)$  of the system.【計分：10分】



2. A unity feedback control system is shown in Fig. B. (1) Determine the range of gain  $K$  for stability.【計分：3分】 (2) Sketch the root loci of the control system.【計分：6分】 (3) Verify that the formulas of describing three root-locus branches are straight lines.【計分：6分】
3. The voltage equation of a dc motor is written as

$$e_a(t) = R_a i_a + L_a \frac{di_a(t)}{dt} + K_b \omega_m(t)$$

where  $e_a(t)$  is the applied voltage;  $i_a(t)$ , the armature current;  $R_a$ , the armature resistance;  $L_a$ , the armature inductance;  $K_b$ , the back-emf constant;  $\omega_m(t)$ , the motor velocity; and  $\omega_r(t)$ , the reference input voltage. Taking the Laplace transform on both sides of the voltage equation, with zero initial conditions and solving for  $\Omega_m(s)$ , we get

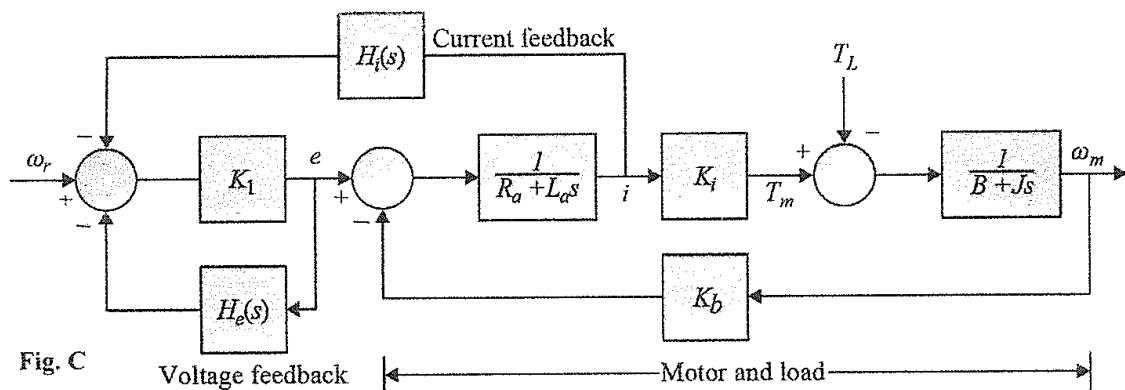
$$\Omega_m(s) = \frac{E_a(s) - (R_a + sL_a)I_a(s)}{K_b}$$

which shows that the velocity information can be generated by feeding back the armature voltage and current. The block diagram in Fig. C shows a dc-motor system, with voltage and current feedbacks, for speed control.

- (1) Find the transfer functions  $\left. \frac{\Omega_m(s)}{\Omega_r(s)} \right|_{T_L=0}$  and  $\left. \frac{\Omega_m(s)}{T_L(s)} \right|_{\Omega_r=0}$ , respectively.【計分：6分】

- (2) Let  $K_1$  be a very high gain amplifier. Show that when  $H_i(s)/H_e(s) = -(R_a + L_a s)$ , the motor velocity  $\omega_m(t)$  is totally independent of the load-disturbance torque  $T_L$ .【計分：3分】

- (3) Find the transfer function between  $\Omega_m(s)$  and  $\Omega_r(s)$  ( $T_L = 0$ ) when  $H_i(s)$  and  $H_e(s)$  are selected as in part (2).【計分：6分】



4. Fig. D shows a single-link robot driven by a DC servomotor. To amplify the torque  $\tau_m$  generated by the motor, a gear transmission with gear ratio  $n$  (i.e.  $n = \text{teeth number of Gear 2} / \text{teeth number of Gear 1}$ ) is used between the motor shaft and the robot link. Let  $I_m$  be the moment of inertia of the motor shaft and Gear 1 combination, and  $I_L$  be the moment of inertia of the robot link together with Gear 2. A rotational damper of coefficient  $b_m$  is placed at the end of Gear 1 to model the possible viscous friction.

- (a) What is the relation between the motor speed  $\omega_m$  and the link speed  $\omega_L$ , and determine the equivalent moment of inertia referred to the motor shaft. (5%)
- (b) Using  $\tau_m$  as the control input and the rotation angle (say,  $\theta$ ) of the link as output, obtain the transfer function describing the dynamic relation between the torque  $\tau_m$  and the output angle  $\theta$ . (5%)
- (c) If  $I_m = 0.01 \text{ Kg}\cdot\text{m}^2$ ,  $I_L = 1 \text{ Kg}\cdot\text{m}^2$ ,  $n = 10$ ,  $b_m = 0.001 \text{ Kg}\cdot\text{m}^2/\text{s}$ , determine the system type and find the impulse response and step response of the single-link robot respectively (15%)

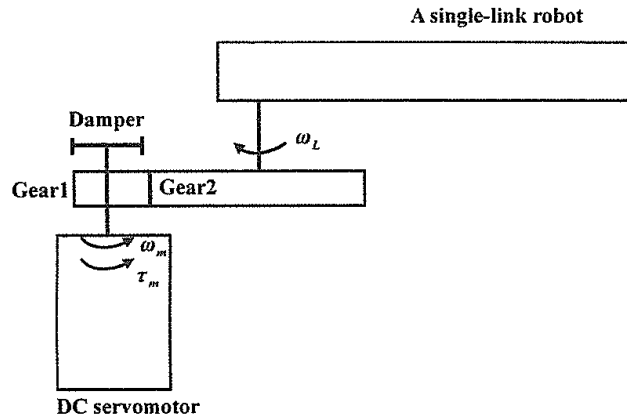


Fig. D

5. Consider a second order system  $G(s)$  (in Fig. E), of which the nominal value of the parameter  $A = 0.1$ . Please design a feedforward gain  $K$  and PD controller so that the response of the closed loop system can satisfy the **both** specifications listed below: 【計分：15分】

- (1) rising time  $\leq 0.1 \text{ sec}$ , overshoot  $\leq 10\%$ , steady state error  $\leq 5\%$  for the unit step reference command  $r$
- (2) disturbance rejection for the unit step  $w$  to 5% within 0.1 sec.

If the parameter  $A$  varies between 0.05–0.15, can you design a controller so that the closed loop system can still achieve the above specifications? 【計分：10分】

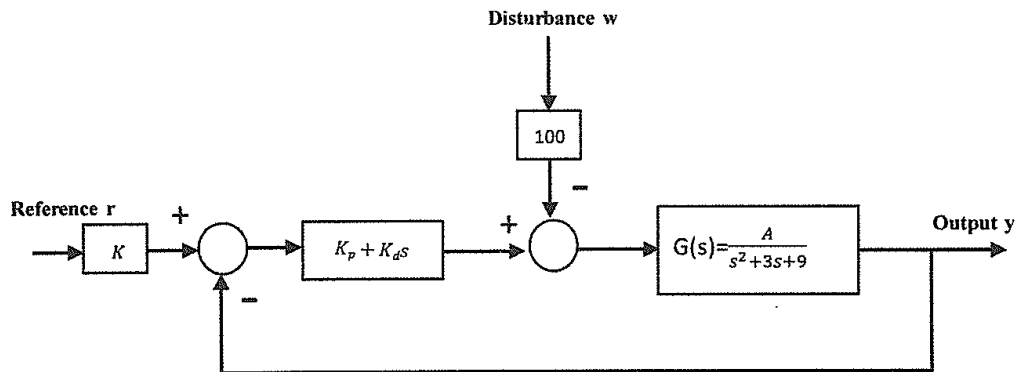


Fig. E