☑ 答案請務必填在電腦閱卷之「答案卡」上。

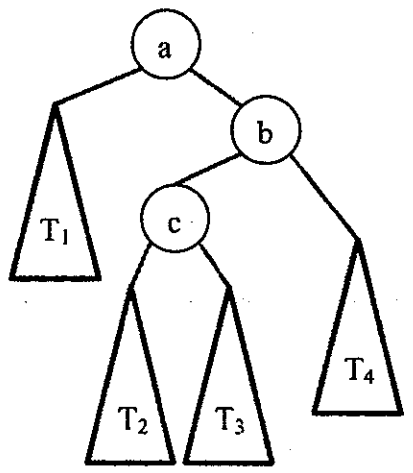
(一) 是非題 (共 10 小題，若你覺得該小題命題正確，請填 (A)，錯誤，請填 (B)。每題答對得 4 分，答錯倒扣 4 分，未填答者不計分也不扣分，倒扣至是非題總分為 0 分為止)

1. If a computer program has the worst case runtime of n*n!, where n is the program input size, then the program has the runtime complexity $O(n^n)$.
2. The data in a linked list are usually connected by "pointers" in implementation. Therefore, its "find(Data d)" operation (i.e. to find whether the data 'd' is in the linked list) is usually more efficient (in terms of runtime) than the "find(Data d)" operation of an array.
3. Even though the minimum data of a min-heap is always on the root of the tree and it takes O(1) time to find it, the removal of this minimum data may take O(log n) time as we need to arrange the nodes to maintain the tree structure.
4. A binomial heap is composed of at most O(log n) binomial trees. Therefore, it takes O(log n) time to find an arbitrary data in the heap.
5. A splay tree is a specially designed balanced binary search tree so that it takes amortized O(1) time to insert new data to the tree.
6. A red-black tree is highly balanced -- the path from its root to the farthest leaf is no more than twice as long as the path from the root to the nearest leaf.
7. Given an arbitrary directed acyclic graph (DAG) of n nodes, it takes $\Omega(n)$ time to find the node with the minimum data (assuming the comparison of the data of two nodes has constant time complexity).
8. For an undirected graph G, the *clique number* $\omega(G)$ is the cardinality of its largest clique, and the *chromatic number* $\chi(G)$ is the minimum number of colors needed to color the vertices, such that no edge has end-points with the same color. We have $\omega(G) \leqq \chi(G)$.
9. If it takes O(1) time to find the minimum node of a heap, then it must take O(n) time to find the maximum node of this heap.
10. Given two leftist heaps of n and 1 nodes, it takes O(1) time to combine them into a new leftist heap. Therefore, the insertion operation of leftist heap is also O(1).

見背面

(二) 複選題 (共六小題，每題 10 分, 題內每個選項單獨計分，答對得 2 分，答錯倒扣 2 分，倒扣至複選題總分為 0 分為止。如某題未作答則該題得 0 分，不到扣。)
(例：正確答案為 ABC，若答 BCD，則答對三個選項(BCE)，答錯兩個選項(AD)，故共計得 3*2 - 2*2 = 2 分)

11. Given an AVL tree as shown below, let T_1, T_2, T_3 and T_4 be full binary sub-trees with heights equal to *h*, *h*-1, *h*-1, and *h*, respectively. If we insert a series of nodes orderly to the leaves of T_4, T_3, T_2, and T_1, one node to each sub-tree at a time, then after rotation(s) and re-balancing, which of the following is(are) true?



(A) There are totally two single rotations encountered during these insertions.
(B) Node 'c' will be the root of the resultant AVL tree.
(C) 'b' will be the parent of 'a'.
(D) The roots of the sub-trees T_2 and T_3 are sibling nodes of a common parent.
(E) If we insert another node to the leaf of T4, the AVL tree will become unbalanced again.

12. Given a set of data of size n, let an unbalanced binary search tree be built by randomly inserted these data to the tree, one at a time. Which of the following is(are) true?

(A) For n large enough, the probability to obtaining a tree with height of $\lceil \log_2 n \rceil$ is more than 0.5. (note: $\lceil\ \rceil$ is the ceiling operation)
(B) The amortized time complexity of inserting a new data to the resultant tree is O(log n).

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(C) Let n be an exponential of 2 and let $n = 2^k$. If we randomly insert another set of data of size n to this tree, then the depth of any of its leaf nodes must be equal to or greater than 2*k.
(D) It takes O(n) to find the minimum data.
(E) It takes O(n log n) to make this unbalanced binary search tree balanced.

13. Inserting nodes to a 2-3-4 tree with the following order using top-down insertion: { 15, 14, 13, 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 }. Which of the following is(are) true?

(A) The root node of the resultant tree is a 3-node.
(B) There is only one 4-node in the resultant tree.
(C) Numbers 9 and 10 are in the same node.
(D) More than half of the nodes are 3-node.
(E) If we insert a new number "16" to the tree, the height of the tree will be incremented by one.

14. Let the `struct OPair` store an ordered pair of numbers as follows:

```
struct OPair {
   int _first;
   int _second;
};
```

Let `n(a, b)` denote an object of this type, where `n` is the name of the object, and `a` and `b` are the `_first` and `_second` data members of `n`. Sometimes we may just use `(a, b)` to represent the pair of numbers when the name of the object is not of concern.

Given a hash of `OPair` with number of buckets equal to 100 and with the hash function defined as:

```
int hashFunction(const OPair& n)
{
    int t = n._first;
    t += ((n._second << 8) % 37);
   return (t % 100);
}
```

Clearly, given an `OPair` object `n`, this function will return the index of bucket for inserting the object to the hash (i.e. between 0 and 99). For example, `hashFunction((12, 34))` should return 21.

見背面

Which of the following statement(s) is(are) true?

(A) `hashFunction((100, 100))` returns 37.
(B) `hashFunction((55, 66))` is smaller than `hashFunction((94, 87))`.
(C) `hashFunction((12345, 0))` is smaller than `hashFunction((23456, 37))`.
(D) Inserting `(55, 66)` and `(66, 55)` will introduce a collision.
(E) If we change the number of buckets to 200, the truthfulness of the above 4 statements remains the same since the hash function does not change.

15. Let H_a and H_b be two binomial heaps with numbers of nodes n_a and n_b, respectively. Let's compose them into a new binomial heap H_c. Which of the following is(are) true?

(A) If n_a is equal to n_b, then the composed H_c contains a binomial tree of order 0.
(B) The composition takes $O(\log(n_a + n_b))$ time.
(C) H_c contains at least $(\lfloor \log_2(n_a + n_b) \rfloor + 1)$ binomial tree(s), where "$\lfloor\ \rfloor$" is the floor operation.
(D) H_c must contain a binomial tree of order $\log_2(\max(n_a, n_b))$.
(E) Let r_a and r_b be the minimum nodes of H_a and H_b, respectively. Then both of r_a and r_b must be roots of some binomial trees in H_c.

16. Which of the following statement(s) about "graph" is(are) true?

(A) A cube is a plannar graph.
(B) The complement graph of a rectangular has the maximum clique of size 2.
(C) For a non-empty directed graph, there must exsit one and only one strongly connected component (SCC).
(D) A connected component of a graph must contain at least half of its vertices.
(E) If we partition a graph into n cliques, then we must be able to find a subset of at most n vertices in which no two vertices in this subset are adjacent.

試題隨卷繳回