※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. (22%) Consider a pulse amplitude modulation system where a waveform $s(t) = s_1(t) = g(t)$ is transmitted if the source bit is 1 and $s(t) = s_2(t) = -g(t)$ if the source bit is 0. Suppose the signal passes through an additive white Gaussian noise (AWGN) channel and the received signal at the receiver is

$$x(t) = s(t) + w(t)$$

where $w(t)$ is a random Gaussian process with a power spectral density $S_W(f) = N_0/2, \forall f$. At the receiver, a matched filter $h(t) = g^*(-t)$ ("*" denoting complex conjugate) is applied to the received signal $x(t)$, resulting in $r(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$ (Note: without loss of generality, here we allow the transmit and receiver filters to be non-causal). We further assume the signal $g(t)$ has energy $E_s = \int_{-\infty}^{\infty} |g(t)|^2 dt$.

   (a) (8%) Sampling $r(t)$ at time $t = 0$, we obtain the sample $r(0)$. Express the sample $r(0)$ as a sum of a signal term and a noise term. Find the signal-to-noise ratio at $r(0)$ in terms of E_s and N_0.

   (b) (7%) Suppose the receiver decides that the transmitted bit is 1 if $r(0) > 0$ and 0 otherwise. Denote η as the signal-to-noise ratio. If a bit "1" was transmitted, find the probability that the receiver gives a wrong decision (bit 0). Write your answer in terms of η and using the Q-function. Recall that the Q-function is defined by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-u^2/2} du.$$

   (c) (7%) If the source bit is 0 with a probability p and 1 with a probability $1-p$. Find the overall bit error rate of the system in terms of η and p.

2. (18%) The root-mean-square (rms) bandwidth of a signal $g(t)$ of finite energy is defined by

$$W_g = \left(\frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2 df}{\int_{-\infty}^{\infty} |G(f)|^2} \right)^{1/2}$$

where $G(f)$ is the Fourier transform of $g(t)$ defined by

$$G(f) = \int_{-\infty}^{\infty} g(t) \exp(-j2\pi ft) dt.$$

Similarly, the root-mean-square (rms) duration of the signal g(t) is defined by

$$T_g = \left(\frac{\int_{-\infty}^{\infty} t^2 |g(t)|^2 dt}{\int_{-\infty}^{\infty} |g(t)|^2} \right)^{1/2}.$$

   (a) (7%) Show that $T_g W_g \geq \frac{1}{4\pi}$.

   (b) (5%) Give an example of $g(t)$ such that the equality in (a) holds (no explanation needed).

   (c) (6%) Find the necessary and sufficient conditions for the equality in (a) to hold.

3. (5%) Which of the following is true? (Each correct choice gets 1 point; each incorrect choice gets -1/2 points; leaving the whole problem blank gets zero points).

   (A) Compared to the amplitude modulation (AM), the double-side band suppressed carrier (DSB-SC) modulation scheme saves both bandwidth and power.

   (B) Coherent BPSK and QPSK have the same bit error rate performance.

   (C) Delta modulation may suffer from a slope overload distortion if the message signal varies too fast.

   (D) In pulse-code modulation, if the sampling frequency is greater than twice the maximum frequency content of the message signal, then the quantization error can be reduced to zero.

   (E) None of the above.

4. (5%) Which of the following is true?

   (A) In a non-coherent receiver, the carrier phase synchronization can be inaccurate but the knowledge of the symbol timing should still be available.

   (B) Compared with the ideal Nyquist channel, the raised-cosine spectrum requires a greater bandwidth resource, but is less sensitive to symbol timing errors.

   (C) An equalizer at the receiver can be used to remove or mitigate the effect of inter-symbol interference.

   (D) In an M-ary discrete PAM system, applying a gray code may help improve its bit error rate performance, but not the symbol error rate performance.

   (E) None of the above.

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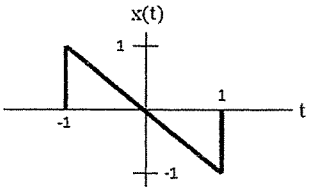
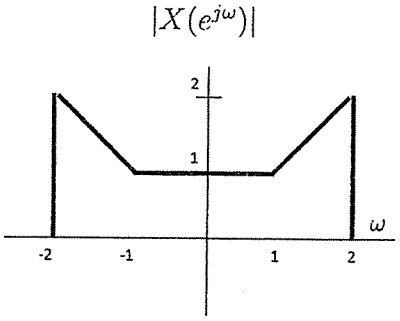
5. Calculating the Fourier transform $X(j\omega)$ of the signal $x(t)$ below (9%).

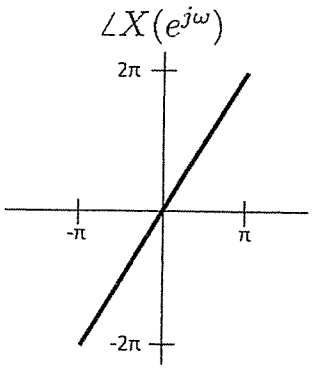


6. Suppose we are given the following facts about a sequence $x[n]$:
   - $x[n]$ is periodic with period $N = 6$.
   - $\sum_{n=0}^{5} x[n] = 3$
   - $\sum_{n=1}^{6} (-1)^n x[n] = 2$
   - $x[n]$ has the minimum power per period among the set of signals satisfying the preceding three conditions.

   Please find the sequence $x[n]$ (12%).

7. Consider the sequence $x[n]$ whose Fourier transform $X(e^{j\omega})$ is depicted for $-\pi \leq \omega \leq \pi$ as below. For each of the following statements, please determine a statement is **true**, **false** or **uncertain** (due to insufficient information).
   (a) $x[n]$ is periodic (3%).
   (b) $x[n]$ is real (3%).
   (c) $x[n]$ has finite energy (3%).





8. Consider a stable and causal system with impulse response $h(t)$ and system function $H(s)$. Suppose $H(s)$ is rational, contains a pole at $s = -3$, and does not have a zero at the origin. The location of all other poles and zeros is unknown. For each of the following statements, please determine a statement is **true**, **false** or **uncertain** (due to insufficient information).
   (a) $\int_{-\infty}^{\infty} h(t)dt = 0$ (2%)
   (b) The Fourier transform of $h(t)e^{5t}$ converges. (2%)
   (c) $\frac{dh(t)}{dt}$ contains at least one pole in its Laplace transform. (2%)
   (d) $h(t)$ has finite duration (2%)
   (e) $H(s) = H(-s)$ (2%)
   (f) $th(t)$ is the impulse response of a causal system. (2%)

9. Consider a stable and causal system with impulse response $h[n]$ and system function $H(z)$. Suppose $H(z)$ contains a pole at $z = \frac{1}{3}$, and a zero somewhere on the unit circle. The location of all other poles and zeros is unknown. For each of the following statements, please determine a statement is **true**, **false** or **uncertain** (due to insufficient information).
   (a) The Fourier transform of $3^{-n}h[n]$ converges. (2%)
   (b) $H(e^{j\omega}) \neq 0$ for all ω (2%)
   (c) $h[n]$ has finite duration (2%)
   (d) $h[n]$ is real (2%)

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