1. (30%) Assume that a linear time-invariant (LTI) system has a transfer function given by $H(s) = (s-1)/[(s+2)(s-3)]$

(a) (5%) Find the region of convergence (ROC) for H(s) if the LTI system is causal and unstable.

(b) (5%) Find the ROC for H(s) if the LTI system is noncausal and stable.

(c) (5%) Find the ROC for H(s) if the LTI system is anticausal and unstable.

(d) (5%) Find the unit impulse response h(t) for the LTI system of Part (a).

(e) (5%) Find the unit impulse response h(t) for the LTI system of Part (b).

(f) (5%) Find the unit impulse response h(t) for the LTI system of Part (c).

2. (20%) Assume that a linear time-invariant (LTI) system has a transfer function given by $H(z) = (2z-5/2)/[(z-1/2)(1-2z^{-1})]$

(a) (5%) Find the region of convergence (ROC) for H(z) if the LTI system is causal and unstable.

(b) (5%) Find the ROC for H(z) if the LTI system is noncausal and stable.

(c) (5%) Find the unit impulse response h[n] for the LTI system of Part (a).

(d) (5%) Find the unit impulse response h[n] for the LTI system of Part (b).

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3. (50%) Consider a baseband communication system shown in Figure 1. Here the pulse shaping filter $p(t)$ is real-valued and is usually chosen as a well-localized signal both in time and frequency domain. The source signal $s[n]$ is a WSS zero-mean discrete-time signal whose autocorrelation function is $\mathbb{E}[s[n]s^*[n+m]] = \delta[m]E_s$. The transmitted signal $s(t)$ is defined as

$$s(t) = \sum_{m=-\infty}^{\infty} s[m]p(t-mT).$$

The received signal is $r(t) = s(t) + w(t)$ where $w(t)$ is a WSS zero-mean additive white Gaussian noise whose power spectral density is $\frac{N_0}{2}$.

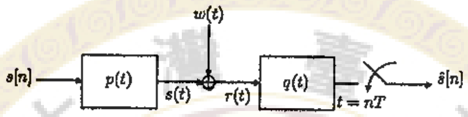

Figure 1: Baseband communication system

We would like to design the receive filter $q(t)$ under some constraints. The detected signal $\hat{s}[n]$ is obtained by sampling the signal at the receive filter output:

$$\hat{s}[n] = \left[\int_{-\infty}^{\infty} q(\tau)r(t-\tau)d\tau\right]_{t=nT}.$$

Both $p(t)$ and $q(t)$ are not necessarily causal.

(a) (10%) Suppose the desired frequency domain of the pulse $p(t)$ is a rectangular function:

$$P(f) = \begin{cases} T, & |f| \le \frac{1}{2T} \\ 0, & \text{otherwise} \end{cases}.$$

What is the time-domain pulse-shaping function $p(t)$?

(b) (12%) Suppose the desired frequency domain of the pulse $p(t)$ is a raised-cosine function with a roll-off factor α, $0 < \alpha < 1$, defined as

$$P(f) = \begin{cases} T, & |f| \le \frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1+\cos\left(\frac{\pi T}{\alpha}\left[|f| - \frac{1-\alpha}{2T}\right]\right)\right], & \frac{1-\alpha}{2T} < |f| < \frac{1+\alpha}{2T} \\ 0, & \text{otherwise} \end{cases}.$$

What is the time-domain pulse-shaping function $p(t)$?

(c) (8%) In (b), with a larger roll-off factor α, will the pulse shaping filter $p(t)$ be more time-localized, or the other way around? Please explain your answer.

(d) (10%) In order to maximize the signal-to-noise ratio of $\hat{s}[n]$, what is the best choice of $q(t)$? Please write your answer in terms of $p(t)$.

(e) (10%) In order to minimize the mean square error $\mathbb{E}[||\hat{s}[n] - s[n]||^2]$, what is the best choise of $q(t)$? Please write your answer in terms of $p(t)$, N_0, and E_s.

試題隨卷繳回