國立臺灣大學 102 學年度碩士班招生考試試題

1. (15 points) Prove or disprove that for every positive integer n,

$$C(3n,n) = \sum_{k=0}^{n}[C(n,k)\cdot C(2n,k)],$$

where $C(n,k)$ is the coefficient of the x^k term in the expansion of $(1+x)^n$

2. (15 points)In an election with two candidates A and B, if candidate A receives p votes and candidate B receives q votes with $p > q$, what is the probability that A will be strictly ahead of B throughout the count?

3. (15 points) Solve the following recurrence:

$$\begin{aligned} a_0 &= 7, & (1)\\ a_1 &= 13, & (2)\\ a_n &= 2a_{n-1} - a_{n-2} + 2, \text{ for } n \geq 2. & (3) \end{aligned}$$

4. (20 points) Given the relation $R = \{(a,b) | a \in \mathbb{Z}, b \in \mathbb{Z}, a > b\}$ on the set of integers $\mathbb{Z}$. Find
   (a) The symmetric closure of R.
   (b) The transitive closure of R.

5. (15 points) Given a graph G with n vertices. Let s, t be two vertices such that any path from s to t contains at least $\lfloor \frac{n}{2} \rfloor + 1$ edges. Prove that any two paths from s to t share a common vertex other than s or t.

6. (20 points) Given any integer $n > 2$ and any sequence of n positive integers $(d_1, d_2, \ldots, d_n)$ whose sum is exactly 2n -2. Prove that there exists some tree T which has this sequence as its degree sequence.

試題隨卷繳回