國立臺灣大學101學年度碩士班招生考試試題

1. Suppose that $E(X)=5$ and $E[X(X-1)]=27.5$ What is **(10%)**
   (a) $E(X^2)$? (b) $V(X)$?
2. Compute the following binomial probabilities directly from the formula for $b(x;n,p)$. **(10%)**
   (a) $b(3;\ 8,\ 0.6)$ (b) $P(3 \le X \le 5)$ when $n=8$ and $p=0.6$
3. The flow rate $y(m^3/\text{min})$ in a device used for air quality measurement depends on the pressure drop x (inch of water) across the device's filter. Suppose that for x values between 5 and 20, the two variables are related according to the simple linear regression model with true regression line $y=-0.12+0.095x$. **(20%)**
   (a) What is the expected change in flow rate associated with a 1-inch increase in pressure drop?
   (b) What change in flow rate can be expected when pressure drop decreases by 5 inches?
   (c) What is the expected flow rate for a pressure drop of 10 inches?
   (d) Suppose that the standard deviation of the flow rate is $\sigma_y = 0.025\ m^3/\text{min}$, and consider a pressure drop of 10 inches. What is the probability that the observed value of flow rate will exceed $0.835\ m^3/\text{min}$?
4. A random variable X is distributed with the following gamma density with parameter $(\alpha=40,\beta)$. **(20%)**
   $$f_X(x;\alpha,\beta)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta},\ x\ge 0,\ \alpha>0,\ \beta>0$$
   A random sample of size 10 from the above density resulted in a sample mean of 82. Calculate the maximum likelihood estimator of β based on these observed data.
5. The moment generating function of a gamma random variable X (density function shown in problem 4) can be expressed by the following equation. **(20%)**
   $m_X(t)=(1-\beta t)^{-\alpha}$ for $t<1/\beta$.
   The exponential distribution is a special case of the gamma distribution with $\alpha=1$. Prove that the sum of n independent identically distributed exponential random variables with a common parameter β is a gamma random variable with parameters $\alpha=n$ and β.
6. A random sample of size $n=100$ is taken from a population with unknown mean μ and standard deviation $\sigma=5$ grams. The sample mean is $\bar{x}=28.4$ grams. Conduct the hypotheses test $H_0:\mu\ge 30$, $H_1:\mu<30$ at level of significance $\alpha=0.05$. **(20%)**

Table of cumulative probability for standard normal distribution Z ($P(Z\le z)=p$)

z	1.05	1.15	1.25	1.35	1.45
p	0.8531	0.8749	0.8944	0.9115	0.9265
z	1.55	1.65	1.75	1.85	1.95
p	0.9394	0.9505	0.9599	0.9678	0.9744
z	2.05	2.15	2.25	2.35	2.45
p	0.9798	0.9842	0.9878	0.9906	0.9929
z	2.55	2.65	2.75	2.85	2.95
p	0.9946	0.9960	0.9970	0.9978	0.9984