1. The following set of laws of electromagnetics have been discovered before Maxwell:

$$(i)\nabla\cdot\vec{E}+\frac{1}{\epsilon_0}\rho=0 \text{ (Gauss's law)}, (ii)\nabla\cdot\vec{B}=0 \text{ (No Name)},$$

$$(iii)\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \text{ (Faraday's law)}, (iv)\nabla\times\vec{B}=\mu_0\vec{J} \text{ (Ampere's law)}.$$

(a) Are these formulas consistent with each other ? If not, then find out the fatal inconsistency as Maxwell (1864) had done some time ago and give the correct and consistent set of formulas governing the electromagnetics. (10 points)

(b) After (a), derive the wave equation for $\vec{E}$ and $\vec{B}$. (15 points)

Note: Here $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, ρ is the charge density and $\vec{J}$ is the current. Also, $\epsilon_0=8.85\times10^{-12}C^2/(N\cdot m^2)$ and $\mu_0=4\pi\times10^{-7}N/A^2$ are the permittivity and permeability of free space, respectively.

2. Conducting Circular Cylinder in Uniform Electric Field. Consider an infinitely long uncharged conducting circular cylinder placed in a uniform electric field E_0 with its axis at right angles to the line of force. Denote the radius of the cylinder by a, and take the x-axis in the direction of of the field and the z-axis along the axis of the cylinder.

(a) Find the potential in the region outside the cylinder. (12 points)

(b) Find the components of the electric field. (8 points)

(c) Find the charge per unit area of the conducting surface. (5 points)

(Hint: You may need to use the cylindrical harmonics functions.)

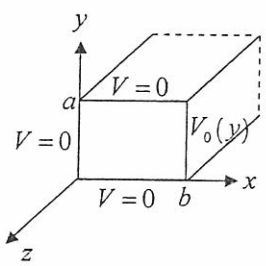
Problem 3 (25%)

A rectangular pipe, running parallel to the z-axis (from $-\infty$ to ∞), has three grounded metal sides at $y=0$, $y=a$, and $x=0$. The fourth side, at $x=b$, is maintained at a special potential $V_0(y)$.

(a) Develop a general formula for the potential $V(x,y)$ within the pipe. (10%)

(b) Find the potential explicitly, for the case $V_0(y)=V_0$ (a constant). (10%)

(c) If $a=b$ and $V_0=1$, what is the *numerical* value of V at the center point $(b/2, a/2)$? (5%)



Problem 4 (25%)

A sphere of radius R, filled with a linear dielectric material with the relativity permittivity ε_r, is placed in an otherwise uniform electric field $E_0\hat{z}$.

(a) Find the electric field $\mathbf{E}(r,\theta)$ *inside* the sphere. (10%)

(b) Find the electric field $\mathbf{E}(r,\theta)$ *outside* the sphere. (10%)

(c) Determine the polarizability α associated with the dielectric sphere. (5%)