1. **(15%)** Consider a matrix $\mathbf{M} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$. Under what condition is this matrix singular?

2. **(15%)** Find the volume of the parallelepiped spanned by the three vectors in the Cartesian space **a** = (-2, 3, 1), **b** = (0, 4, 0), and **c** = (-1, 3, 3).

3. **(15%)** The expected value is the mean value of the probability distribution. Determine the expected value for the number of heads when a coin is tossed four times.

4. **(15%)** Find the solution for $y'' + y' - 2y = 0$ with the initial conditions of $y(0) = 4$ and $y'(0) = 1$.

5. **(20%)** The function V at a particular point in space is given by: $V = 2x^2 + 4y^2 + 6z^2$. (a) Find the rate of change of V at P(2, -2, 1) in a direction towards (0, 0, 0). (b) Find the magnitude and direction of the maximum rate of change of V at P.

6. **(20%)** Find an orthogonal matrix **C** such that $\mathbf{C}^{-1}\mathbf{AC} = \mathbf{D}$ where **D** is a diagonal matrix with diagonal elements equal to the eigenvalues of **A** where the matrix **A** is given by $\mathbf{A} = \begin{bmatrix} 7 & -2 & 1 \\ -2 & 10 & -2 \\ 1 & -2 & 7 \end{bmatrix}$.

試題隨卷繳回